Forefront
Fractional Reasoning Assessments
2024

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Introduction

Proficiency with fractions at the end of the elementary school years has been demonstrated to be a strong predictor of success in middle and high school (Siegler, 2012). Understanding fractions is also one of the most ambitious goals of the intermediate grades in mathematics (Braithwaite, 2017, Gabriel, 2013). For many students, learning fractions marks the moment when mathematics becomes completely unintelligible, when number sense becomes number nonsense, and teachers instruct students in "copy dot flip," and the use of "butterfly methods." "Ours is not to reason why, just invert and multiply," as the old adage goes.

"As one encounters fractions, mathematical content takes a qualitative leap in sophistication." (Lamon, 2020) Fractions are complex numbers and pose a variety of difficulties for students. For students who are still in the process of conceptualizing place value and who are grappling with multiplicative reasoning, fractions represent a new realm of number ideas that behave very differently from the whole numbers they have encountered up until now. While whole numbers each have a distinct value, fractions represent the same values in different ways (e.g. 3/4 , 6/8, 9/12, etc.) Meanwhile fractions refer to a sometimes not obvious whole and cannot be fully understood without the concept of the whole of which they signify a part. To add even further complexity, the whole may be a set (e.g. a package of 12 cookies.) Making it so that a fraction like ¾ might indicate 9 cookies. Understanding fractions is difficult, seeking to understand how students understand fractions is fascinating.

Various theoretical frameworks for how students progress in their learning of fractions have been proposed. There are state standards, and an evolving research base to help us understand the sequences, stages, progressions, and/or trajectories of learning as students become increasingly competent in their use of fractions (Petit, 2023, Wilkins & Norton, 2018, Fosnot & Dolt, 2002, Battista, 2012, Empson& Levi, 2011, Hackenberg, 2016, Lamon, 2020). The Fractional Reasoning Assessment system presented here is an attempt to provide teachers with a useful tool that at once acknowledges and embraces that complexity, while primarily providing an instructionally helpful tool for teachers.

Fractional reasoning is a rigorous goal for learning. It requires that students can apply their knowledge, communicate their understanding, think flexibly, and achieve computational fluency. When students can reason with fractions, prior knowledge of fractions transfers and is applicable to new learning since students are building on an existing conceptual framework. For students

who depend on rote computations with fractions without clear understanding of why those computations work, each new thing to memorize is independent and disconnected.

Defining Fractional Reasoning

What is meant by fractional reasoning? Fractional reasoning is a specific type of mathematical reasoning. In their 2011 book *Focus in High School Mathematics*, Dick & Hollebrands define mathematical reasoning as, "drawing logical conclusions based on evidence or stated assumptions." The ability to reason is more broadly defined as the ability to use logical, rational, and analytical thought. (American Heritage College Dictionary, 1997).

For these assessments, fractional reasoning is being defined as follows.

> *The ability to apply quantitative understandings of fractions to solve problems, and to communicate and justify thinking verbally, symbolically, through gestures, using visual models, and measurement tools.*
>
> *Fractional reasoning does not exclude the use of procedures, but the rote application of procedures without the ability to explain the logic of those procedures does not constitute fractional reasoning.*

How is fractional reasoning revealed?

Students are asked to compare the fractions $\frac{3}{5}$ and $\frac{4}{9}$.

Student 1
Student 1 says, "Three fifths is more than four ninths because I know that three fifths is more than one half and four ninths is less than one half. I know this because three is more than half of five, and four is less than half of nine."

This kind of response demonstrates fractional reasoning. It shows an understanding of the relationship of the numerator to the denominator. It also shows an ability to compare to the benchmark fraction of one half.

Student 2
Another student might say, "Three fifths is less than four ninths because I multiply 9 x 3 and that is 27 and I multiply 5 x 4 and that is 20." Teacher: "How does that work?" Student: "I imagine butterfly wings on the fractions."

In this second example, the student has not communicated fractional reasoning. They have found the correct answer, but have demonstrated they are using whole number thinking, not an understanding of the logic of *why* three fifths is the larger fraction.

Student 3
A third Student might say, “Three fifths is larger. I found common denominators.”
Teacher: “Tell me more. Why does that work?”

The reasoning will now be revealed in the response of the student. If the student says something like, “Because common denominators means the pieces are the same size,” this displays some reasoning. If the student simply explains the procedure, or just says, “It’s cross multiplication,” or something like that, it shows that their ability to truly *reason* with fractions is still developing.

All three students arrived at the correct answer, but not all three demonstrated a true ability to reason with fractional numbers.

The assessment of reasoning, therefore, must look beyond correct and incorrect answers for evidence of how students understand fractions. This necessitates that teachers pay attention to what students are communicating through their words, drawings, and gestures.

Asset-based Assessment

Asset-based assessments are assessments that help to reveal what a student knows as much as, if not more than, what they still need to learn. If this assessment is successful it will make visible early and perhaps flawed understandings of fractions for teachers to be able to build from and connect new learnings.

For the screener assessments this means that teachers will need to be attentive and occasionally interact with the students as the students solve the problems. They should watch students for indications of how they are solving the problems. When there are students who are not writing anything, teachers should encourage them to attempt the problems. Where there are students who give answers without evidence of how the problem was solved, teachers should ask them to express their reasoning. The purpose of this assessment is to get as much information as efficiently as possible about how students understand fractions.

When the screener assessment is not sufficient for teachers to understand what the student does know, teachers are encouraged to use tasks from the diagnostic assessment. While the screener parts of this assessment can be administered to the whole class, the diagnostic assessment is completed as an individualized interview. Through observing, listening to the student, and asking questions, the teacher is able to reveal the students’ reasoning. Through interacting with the students, teachers are able to recognize in more detail specific starting points for instruction and discern assets the student brings with them to guide their instructional methods and sequences.

What are Screening and Diagnostic Assessments?

The Fractional Reasoning Screeners are designed to:

- Help educators understand, in general, the fractional reasoning of students and groups of students.
- Help educators identify specific topics for targeted instruction and practice.
- Efficiently and accurately help educators identify students who would benefit from small group instruction and targeted supports.
- Help schools and school systems monitor tier 1 progress and answer questions like, “Are we improving in our instruction of the ideas aligned with the screener?” and “Do we see the impact of our efforts and initiatives?”

The Fractional Reasoning Diagnostic Assessments can be used to:

- Probe the understanding of students for whom the teacher was curious about after evaluating the results of the FRS Assessments.
- Reveal inchoate understandings
- Inform goalsetting and planning for instruction by helping teachers to identify prior understandings from which to build new ideas.

The Administration of the Fractional Reasoning Assessments

Fractional Reasoning Screeners
Grade level screening assessments are designed to be administered to whole groups of students and range from 2 - 4 pages in length. They are intended to take approximately 25 - 45 minutes to administer. With this release, there are 3 screening assessments intended to align with end of year expectations for USA grades 3, 4, and 5. The idea is that they will be used before or early in the instruction in fractions in grades 4, 5, and 6. However, teachers may utilize them as needed for their purposes (e.g. at the conclusion of fractions instruction to identify needs areas for reteaching or for progress monitoring purposes.)

Fractional Reasoning Diagnostic
The diagnostic portion of the assessment is administered as an interview assessment with individual students. The amount of time necessary to complete the assessment varies widely from 5 - 35 minutes depending on the student. Students who are early in their development of fractional reasoning tend to take less time, while those who already demonstrate a capacity for more sophisticated reasoning tend to take longer.

The Intended Purposes of the Forefront Fractional Reasoning Assessments

Forefront’s Fractional Reasoning Assessments are intended to serve a variety of practical purposes for teachers, schools, and districts.

Screening Assessments: The Fractional Reasoning Screeners are intended to help teachers understand their students’ current understanding in preparation for new learning. Results from the assessment are intended to provide helpful formative information for teachers to use in

planning instruction and strategic grouping of students. The screeners are designed to help teachers understand student readiness for new learning, and to help them understand which facets of fractional reasoning students have developed competency with, and which facets might need additional instruction and practice in preparation for new learning. Similarly, these assessments also help to identify students who would benefit from additional instruction in preparation for grade level content. Where there is universal implementation and systematic data collection, the FRS can be used to monitor systemic improvement.

Formative Assessments: The Fractional Reasoning Assessments are intended to provide rich, formative information and provide insights into student thinking that help teachers identify specific ideas for instruction. They are designed to help teachers understand student thinking so that they can build on that knowledge as they instruct. They are not intended to be used to inform grading. Teachers are encouraged to use the assessment results to provide feedback to students.

Professional Learning: Complementary to the use of the Fractional Reasoning Assessments for formative purposes is the value that these assessments aim to provide for supporting teachers' development of their understanding of how students come to reason with fractions, and the accompanying effective pedagogical practices. This learning can be systematized in a variety of ways: coaching, PLCs, and Data Driven Instruction are three examples.

Recommendation 5 from the IES Practice Guide, *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (NCEE, 2010) centers professional learning as a necessary element for improving outcomes for students. Recommendation 3 states the importance of developing "teacher's ability to assess students' understanding and misunderstanding of fractions." (p.44) The recommendation to "provide teachers with opportunities to analyze and critique student thinking about fractions" (ibid.) is central to the goal of the Fractional Reasoning Assessments.

Progress Monitoring: When any one of the assessments is administered more than once after instruction has taken place, the Fractional Reasoning Screeners can be used to monitor the progress of individuals and groups of students. When the series is used for school-wide and district-wide purposes the assessments can be used to monitor multi-year efforts to improve outcomes with fraction related topics. To this end, the FRS can be used to help determine the effectiveness of the implementation of instructional programs for teaching fractional reasoning.

Family Communication: The assessments are intended to help inform conversations with the families of students to help them understand how their child is developing in their fractional reasoning and to support them with ideas for fostering that development at home. Forefront® users have access to fully customized letters that are automatically generated by the software for each student.

Diagnostic Assessments: In addition to the screening tasks, the FRS provide additional tasks for teachers to utilize when the screening portion of the assessments have not provided sufficient

information for guiding their instructional efforts. The diagnostic is intended to be asset-based, that is, the assessments are designed to help teachers identify fractional reasoning and the extent of understanding.

The Forefront Fractional Reasoning Framework

While there is no universally accepted framework for organizing the developmental milestones of fractional reasoning, several different frameworks were considered in the writing of these assessments. These theoretical frameworks, some of which are driven by evidence from observing and assessing children, and others which arise from an analysis of the mathematics itself, informed task design as well as grade level alignments. (Wilkins & Norton, 2018, Fosnot & Dolt, 2002, Battista, 2012, Empson & Levi, 2011, Hackenberg, 2016, Petit, et. al., 2020, ALCoS, CCSS).

Theoretical frameworks are helpful for understanding student thinking, and they are especially helpful in research. Theoretical frameworks, however, often are misaligned with curricular programs and grade level expectations. In this way, an assessment designed purely around a theoretical framework can be less helpful for informing and guiding classroom practice.

In order to synthesize the important, evidence-based ideas from the research world with the day-to-day realities of classroom instruction, and to create a series of assessments that are both meaningful and manageable, five levels have been defined which synthesize curricular expectations with developmental progressions and learning trajectories.

The Framework and Standards Alignments

Forefront's Fractional Reasoning Screeners help teachers to identify students' levels of understanding and preparedness for curricular and new learnings. To help teachers understand where students are in their learning of fractions, the tasks and corresponding results are broken into 5 levels. To make these assessments as practical as possible, there was a deliberate effort to create alignments to curricular expectations as defined by state standards and courses of study in the USA.

The Levels of the Fractional Reasoning Diagnostic Assessment

Level Pre
Students who do not yet demonstrate an understanding commensurate with Level A are considered to be at the Pre level.

Level A - Partitioning
The initial level of fraction conceptualization is the making of fair shares. At this level students are able to form equal sets of discrete objects, partition geometric shapes, and segment lines into equal sized regions. Included in Level A is knowing the words to identify halves and quarters.

Level B - Unit Fractions (Grade 3)
This level includes demonstrating the ability to reason with unit fractions across a variety of contexts. Students at this level can solve problems related to separating and combining fractional parts of wholes in action. Level B includes the idea that unit fractions can be iterated to create a whole. The topics of Level B align with the early fraction work of 3rd grade.

Level C - Proper Fractions (Grade 3 -4)
Proper fractions are non unit fractions less than and greater than one. Students who perform at Level C show the ability to demonstrate their understanding of how unit fractions are iterated to form fractions and their relative wholes across a variety of representations and contexts. This same understanding extends to mixed numbers. Level C also includes the ability to identify and generate equivalent fractions, including conversions of fractions to mixed numbers.

Level D - (End of 5th Grade) Operating with Fractions
The main distinction of Level D is the ability to solve additive and subtractive problems with fractions and to explain their reasoning. Level D of this assessment also includes problems related to the multiplication and division with a whole number and a fractions (unit and proper).

Facets of Fractional Reasoning

Fractional reasoning is multi-faceted. As students deepen their understanding of fractions, they are able to reason with fractions across a variety of modes and contexts. Deep conceptual understanding is evident and grows as students work with a variety of models and metaphors. As Jere Confrey and Alan Malony point out, "An idea unfolds and becomes enriched as one sees various aspects of it and specifically recognizes how it can be refined to make sense of broader related tasks and deeper connections." (2023) No single modality is sufficient for fully demonstrating an ability to reason conceptually with fractions.

Children learn and grow to understand numbers, including fractions in a variety of ways. There are visual models that students work with to describe units and unit fractions. There are words and syntactic forms. There are problem types, like computational problems and comparison problems. And there are problems that involve fractions of sets, and others that involve the partitioning of discrete wholes.

Petit, Laird, Ebby & Marsden (2023) suggest that students' recognition of fractions as numbers, in contrast to whole number reasoning can be revealed when students:

- "locate fractions on a number line
- compare fractions
- identify fractional parts of wholes
- estimate the magnitude of fractions
- operate with fractions"

The assessment system presented here uses a similar set of ideas for distinguishing fractional reasoning.

6 Facets of the Forefront Fractional Reasoning Assessments

The Fractional Reasoning Screeners probe the understanding of fractions across 6 facets.

These same facets are also seen in most common instructional materials, and are explicitly called out in many state standards and curricula.

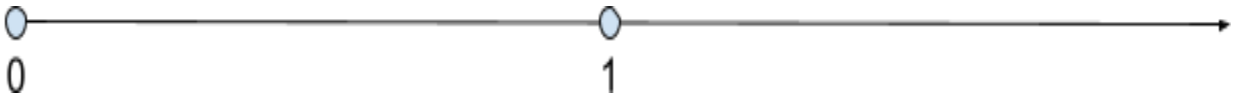
Facet 1: Words and Symbols: This includes being able to name fractional parts as well as being able to read and write fractions. This includes questions like:

What part of the rectangle is shaded? (Level A task)

Facet 2: Shapes: This facet includes the ability to connect symbolic representations of fractions to reasoning with 2 and 3-dimensional shapes: rectangles, circles, rectangular prisms, etc. This includes tasks like, "Here is a rectangle. Show ⅝ of this rectangle?" (Level C task)

Facet 3: Lines: This facet includes the ability to connect fraction numerals to number lines. This includes tasks like:

Where does ⅔ belong on this number line?



(Level C task)

Facet 4: Sets: Sets are discrete, not easily separable items. Pennines, buttons, candies, dots, etc. This facet includes questions like, "I have a package of 12 candies. I want to give ⅓ to my friend. How many should I give them?" (Level B task)

Facet 5: Magnitude, Comparison, and Equivalence

This category of problems at the most basic level includes situations like: Julieta and Jack each have crackers. Jack breaks his cracker in half and says, "Look, I have more! I have 2, you only have 1." Julieta says, "No you don't!" (Level A task)

Who do you agree with and why?

Facet 5 also includes classic comparison problems using <, >, = problems and problems that require finding a common denominator. The assessment of reasoning when comparing necessitates questions that ask how a student knows, and examines whether they can justify their reasoning for why one fraction is greater than another or why they are equivalent.

Facet 6: Computation
Computation includes the combining, separating, or interaction of fractions. At the most basic levels, this includes recognizing which shape is complementary to another for the creation of a geometric figure. At the highest levels of this assessment series, computation involves the addition and subtraction of mixed numbers, the multiplication of whole numbers by fractions, and fractions divided by whole numbers.

The ability to apply reasoning (in contrast with whole number thinking and procedures) to compute with fractions is assessed using contextualized problems. This encourages students to apply their visual thinking and to be less likely to rotely apply a known procedure.

The Overall Structure

Each of the facets is used to elicit thinking across the different Levels, progressing from less complex to more complex as the Levels increase.

The screening assessments work students across a series of tasks approximately at the same Level, depending on their grade level. The tasks provide opportunities for the students to express their fractional reasoning abilities across each of the 6 Facets described above. Therefore, the screening assessments seek to sample performance across each of the facets, moving horizontally, from left to right, across the chart below.

	Words and Symbols	Sets	Shapes and Lines	Words and Symbols	Magnitude, Comparison and Equality	Computation
Pre-Level						
Level A						
Level B						
Level C						
Level D						

While a screening assessment cannot thoroughly assess proficiency across the full spectrum of this framework, tasks across each of the facets help teachers to hone in quickly and efficiently across these different aspects of fractional reasoning.

In contrast to the screening assessments, the Fractional Reasoning Diagnostic is designed to work vertically across each of the facets of the assessment. That is, teachers are encouraged to choose a facet of fractional reasoning and to probe student thinking with questions across that facet, to determine a Level. Teachers may work to determine Levels across each of the Facets, or they may choose one or a smaller set of Facets to focus their attention on for their instructional purposes.

Progressive Formalization and the Fractional Reasoning Assessments

Progressive formalization is an idea that has come from the Freudenthal Institute and Realistic Math Education (Van Reeuwijk, M., 2001, Gravemeijer, K., 2003, Brendefer, J., 2021). Progressive formalization proposes that mathematical ideas, when presented in meaningful contexts, are more accessible, and enable students to engage meaningfully with the problem. When tasks are presented informally, students are more likely to apply their intuitions and reasoning to solve tasks than they would be with more formally presented tasks.

The tasks of the Fractional Reasoning Diagnostic Assessments can be put into three problem types defined by the ideas of progressive formalization: informal, preformal, and formal (Van Reeuwijk, 2001, Gravemeijer & van Galen, 2003, Webb, D. C.2008, Bredefer & Strother, 2021).

Informal, Preformal, and Formal

Informal
Here are two whole crackers. If you break each of these crackers into thirds, how many pieces will you have?

Preformal
Divide each of these rectangles into thirds. How many thirds did you make?

Formal
Solve $2 \div \frac{1}{3}$

While the content of the problem alone, division of a whole number by a unit fraction, when presented formally, is considered 5th grade content according to the CCSS and most state standards, the informal presentation makes this problem accessible to even some 1st and 2nd grade students. Susan Empson and Linda Levin, in their book *Extending Children's Mathematics: Fractions and Decimals* (2011), illustrate in detail how well-designed contexts can make problems accessible to students with a wide range of understanding of fractions.

Contexts and visual models have been used deliberately throughout the Forefront Fractional Reasoning Assessments. This helps teachers to recognize students' ability to engage with the topics at a variety of levels of formality. Students who might be unable to clearly explain their

reasoning with more formally presented tasks are able to reason with similarly structured problems at less formal levels.

Informal Tasks are contextualized and easily accessible. The context provides opportunities for the students to act out, and/or easily imagine the situation. Think of a rectangle. To make this less formal, think of it as a graham cracker. To make it even less formal, provide a paper rectangle for the student to work with. To make it even less formal, use an actual graham cracker. Informal tasks invite intuition, and action. Informal tasks typically include no symbols.

Preformal Tasks use physical and visual models that are teacher created. They are a step toward the abstraction of an idea. Preformal settings are typically adult-invented instructional models and materials like ten-frames, base-ten blocks, abacuses, and fraction pieces. In these assessments, rectangles, circles, and number lines are considered preformal models.

Formal Tasks are generally presented symbolically without context.

While progressive formalization is often thought of as a one-way progression from informal to formal, this is a misconception and with fractions this is most definitely not the case. Quite often we will encounter students who are able to demonstrate some fluency with tasks presented formally (e.g. ¾ + ⅔), yet are unable to match that same problem to a pre-formal task, or are often even less able to imagine an informal application for such a problem. A student who fully demonstrates fractional reasoning can demonstrate their understanding in informal, preformal, and formal ways.

To assist teachers in understanding the use of progressive formalization in their assessment and instructional practices, each of the tasks of the Fractional Reasoning Diagnostic Assessments has been tagged with a level on this model of progressive formalization. While it would be impractical to assess thoroughly all facets of fractional reason across each of the levels of progressive formalization across each of the facets and levels of the FRA, these indicators are intended to provide formative information for teachers to consider how fractional concepts and fractional reasoning can be made accessible to students.

Conclusion

Fractional reasoning is important for all students to develop. The ability to reason with fractions serves as a conceptual springboard for learning algebra and higher mathematics in both middle and high school. As important as this learning is, it is difficult to achieve and to assess. These assessments are designed to inform teacher understanding of their students, and of the topic itself in order to inform and improve instruction and therefore learning outcomes for all students.

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Instructions for the Administration of
The 3/4 Fractional Reasoning Screening Assessment

This assessment is intended to be administered to the whole class at once. The teacher will be asked to read a script in order to establish a pace for the assessment and keep the time demands minimal. There will be students who are unable to complete any of the tasks during the time provided, plan to provide the time at the end of the assessment or at a later time.

When should this assessment be used? This assessment has been designed to be used at one of the three times below.

Mid 3rd Grade: Administer in the midst of 3rd grade fractions instruction as a formative assessment.

End of 3th Grade: Administer the assessment after the conclusion of instruction related to fractions in 3rd grade to identify areas for re-instruction, additional practice, and small group work.

Early 4th Grade: Use this assessment before or early in the instruction of fractions in 4th grade to gather formative information, to identify topics to review, and students who would benefit from pre-teaching before formal instruction with fractions begins.

Time: Plan for 45 **minutes** for the whole group administration and additional time for extended time for targeted students. With some groups the administration of the assessment might be done in as little as 30 minutes.

Preparation:

- Photo copies of the assessment for all students
- Pencils with erasers (not pens) for all students
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the video: There is a video that accompanies this assessment for question 6. Have the video cued before you begin the assessment.

Link to video for #6

Assessment administration:
Please follow the script as written and have a clock or timer ready. This will help to keep the assessments more reliable. It will also ensure that the assessment remains efficient. Students should not be aware of the timing of the tasks. This is not a timed test.

Answer questions only as necessary to ensure students understand the task. Refrain from answering questions that would influence student thinking or provide them clues to how to solve the tasks.

Active Proctoring: Teachers are encouraged to move about the room and to interact appropriately and actively with the students.

- Where there are students who seem to not be engaging, it is appropriate to encourage them, and to re-read the question for them personally to ensure understanding.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning.
- When you see student work that leaves you uncertain about their reasoning, ask them questions and encourage them to write, draw, use number lines, or demonstrate their thinking in other ways.

Forefront users: Forefront has been configured to support data collection for Forefront clients. If you are interested, please reach out to support@forefront.education to request that the assessments be added to your district's resources.

Observations: While systemic collection of observational data is not included in this assessment look-fors are called out in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring: Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the student to ensure the accuracy of the analysis.

Accommodations: Any special accommodations which would normally be given for individual students should also be given for this assessment.

Grade 3/4 Script: Fractional Reasoning Screener

Before you begin the assessment, prepare the video to be shown for Task 6.

Read these instructions completely to familiarize yourself with the instructions in preparation for the assessment passing out the papers. Teachers should read all of the bolded text.

Prepare a clock or timer before you start, but be discreet with it. This is not a timed test, but the watch will help you to keep the students on a reasonable pace as they work through the assessment. The suggested timings are intended to be guidelines; teachers are encouraged to use their best judgment as they lead the students through the assessment. Once almost all of the students have completed a task, continue to the next task and let students know that they will be provided with time to return to unfinished tasks later.

Teacher: **Today we are going to solve some problems to help me to understand how you understand fractions. The results of this assessment will not be part of your grade. This is only to help me to see how you understand fractions and to help me teach you better. For that reason it is important to always explain your answers clearly. Use words, drawings, number lines, and numbers to show your thinking.**

Please keep your eyes focused on your own paper. It is important that I get to know your own thinking about these ideas so that I can support you better. (Some teachers may want to spread students out a bit, or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please do not work ahead, since these questions are a little different than other assessments. The last question will include a video that we will watch together.

If at any time you have a question, please raise your hand. Are there any questions before we get started? (answer questions as necessary)

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

Task 1:

When students are ready, read the script. Pause briefly after you read each of the numbers for Task 1 to give students the opportunity to write their numbers. Continue at a reasonable pace. You may say the numbers more than once if necessary.

Teacher: **For the first question I will read some numbers aloud. Write the numbers in each of the boxes.**

In box A write the number: one half.
In box B write the number: three eights.
In box C write the number: five quarters, or five fourths.

Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Task 2:
Teacher: **Number two has two parts. Part A says, "Shade three fourths of the shapes below. Shade three fourths of the rectangle and three fourths of the circle.**

Pause to allow students to complete part A, then continue to part B.

Find Part B and put your finger on it. (Look for students to find Part B.) **Part B says, "Here is a rectangle. In the space below, draw a rectangle that is four sixths the size of the gray rectangle.**

You may make marks on the gray rectangle to help you. Be sure to show your thinking.

Note: Do not provide, or allow students to use, rulers. If a student asks to use a ruler or some other length measurement device, say something like, **"That's not a bad idea, but just estimate for now. It does not need to be perfect."**

START TIMER FOR 3 MINUTES. Circulate the room to answer questions, check to see that students are getting started, and to ask students to clarify their thinking as necessary.

Once most students have had a chance to complete the task, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to finish."**

Continue to task 3.

Task 3:

Teacher: **Turn the page and find number 3. This question also has two parts. Part A says, Here is a number line with the numbers 0 and 1. Put the numbers one half and one fourth where they belong on the number line.**

Give the students 1-2 minutes to place the numbers on the line.

Find part B. Part B says, "Here is a different number line. It has the numbers 0, 1, and 2 already on it. Put the numbers below onto the number line. (Do not read the fractions.) "**Make a mark where each number belongs, and write the number by the mark."**

Circulate the room to ensure that students are clearly marking their number lines and, if necessary, remind them to write the fractions to indicate which fraction goes with which mark.

5 MINUTES: Allow about 5 minutes for this question (continue along earlier if everyone has finished).

If there are students who have not had enough time, say, **"We are going on to the next question now. If you have not finished, don't worry, I will give you time to work some more later."**

Task 4:
Teacher: **Number four also has two parts. Part A says, "The rectangle with circles below represents a small pack of candies. I want to give one third of the candies to a friend. Show how many of the candies I want to give my friend."**

Include a number in your answer. Circle the candies, or draw them in the space to show how you know.

Give the students 1-2 minutes to complete Part A.

Turn the page and find Part B. Part B says, "The rectangle with circles below represents a pack of 12 candies. I want to give three fourths of the candies to a friend. Show how many candies I want to give my friend."

Circulate to see that students are answering the question. Numeric answers (e.g. 2 candies) are acceptable. If a student writes numeric answers quickly, encourage them to make a drawing or explain how they know.

Give the students about 3 minutes to work on Task 4.

Task 5:
Teacher: **Let's move on to Number 5. If you haven't finished number 4, don't worry, we can come back and finish it later. Use the greater than, equal to, and less than symbols to compare each pair of fractions. Explain your answers using lines, drawings or words.**

The teacher may clarify for students the use of the symbols. If there is any uncertainty, they may ask the student to circle the larger fraction. The goal of this assessment is not to determine whether they can use the symbols accurately, it is to see how students reason about the relative size of the fractions.

Circulate to remind students to explain their reasoning for each comparison. Where students use only numbers to solve (i.e. they find common denominators) ask them to show their thinking with a picture or number line. If students use only shapes to compare the two, ask if they have another way to know that they are correct.

Proceed once everyone has answered, or allow up to 5 minutes. This task tends to take longer than the prior tasks, since students are asked to explain their reasoning about each of the comparisons.

Task 6:
Teacher: **We are going to continue now to number 6. This question includes a short video. First I will read it, and then we will watch a short video that goes with the question. I will tell you when to start after we watch the video.**

"A circle is divided into three equal parts. One part is cut off and removed. How much of the circle remains? Write a fraction and draw to show your answer."

Play the video: *Video Link*

Write a fraction and draw to show how much of the circle remains.

Proceed once everyone has answered, or allow up to 5 minutes. If a student writes a whole number (e.g. 2 parts) ask them if they can write their answer as a fraction.

Conclude the assessment. As you collect the papers, check to see if there are unfinished problems and either keep them for finishing later, or ask the student to complete them. If there were students who needed more time, provide it now, or later. See instructions related to diagnostic assessment for students with more than a few incomplete answers.

3 / 4 Fractional Reasoning - Show what you know.

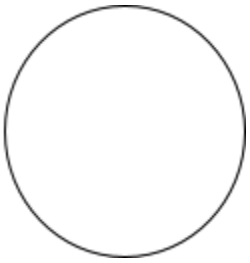
Name _____ Date _____

1. Write the numbers in each of the boxes as your teacher reads them.

A	B	C

2.

Part A: Shade $\frac{3}{4}$ of each of the shapes below.



Part B: Here is a rectangle.

In the space below, draw a rectangle that is $\frac{4}{6}$ the size of the gray rectangle.

3.

Part A: Here is a number line with the numbers 0 and 1.

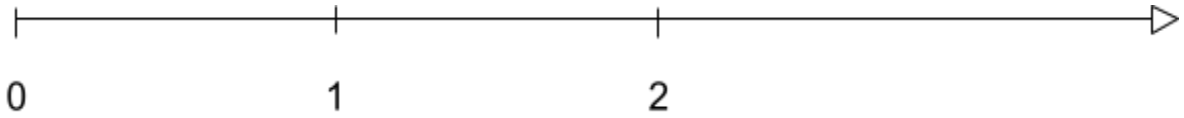
Put the numbers $\frac{1}{2}$ and $\frac{1}{4}$ where they belong on the line.

Part B: Here is a different number line. It has the numbers 0, 1, and 2 already on it.

Put the numbers below onto the number line.

$\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{1}$ $\frac{4}{3}$

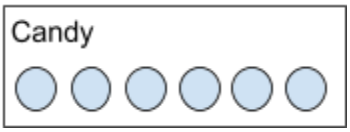
Make a mark where each number belongs, and write the number by the mark.



4.

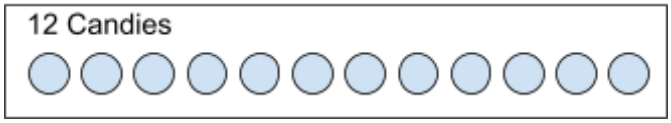
Part A: The rectangle with circles below represents a small pack of candies.

I want to give $\frac{1}{3}$ of the pack to a friend. Show how many of the candies I want to give my friend.



_____ candies

Part B: The rectangle with circles below represents a pack of 12 candies.
I want to give $\frac{3}{4}$ of the pack to a friend. Show how many of the candies I want to give my friend.
Write how many candies on the line below.



_____ candies

5. Use >, =, < to compare each pair of fractions. Make drawings to explain your answers.

$\frac{1}{3}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{3}{4}$ $\frac{3}{3}$ $\frac{5}{5}$

6. Video A circle is divided into 3 equal parts. One part is cut off and removed. How much of the circle remains? Write a fraction and draw to show your answer.

Fractional Reasoning Screener
3/4 Scoring Guide

Forefront data entry: Enter results as instructed in each of the questions.

Standards alignments: At this time, these questions are simply aligned with the 3rd grade domain of fractions - e.g. 3.NF. Specific alignments for different states will be done and configured in Forefront over time.

Fractional Reasoning Lens: Alignments to the Fractional Reasoning Lens are listed with each of the tasks.

Observations: For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These should be taken into consideration during the scoring.

1. Writing Fractions from Dictation FR.3.WS (Words and Symbols)

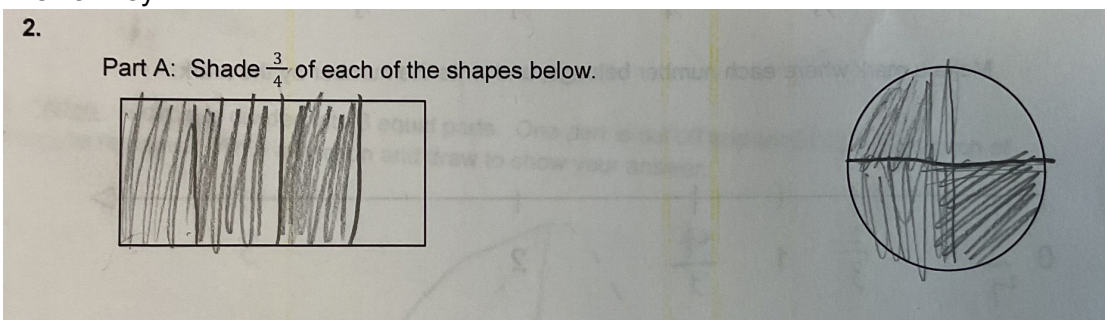
Possible points: 3 One point for each correctly written fraction (enter points in Forefront). Performance Levels: ● All accurate (3 points) ● Not yet (< 3 points)
Answer Key: $A.\ \frac{1}{2}$ $B.\ \frac{3}{8}$ $C.\ \frac{5}{4}$
Clarifications: Credit should be given if the number is legible. While students should be encouraged to write the numerals vertically, give points if they have written them like this: ½.
Readiness expectation: Students should be able to write unit and proper fractions from dictation.

2. Part A: Shade ¾ of Geometric Shapes FR.3.Shapes (Shapes)

Possible Points: 2 One point for each correctly shaded fraction (enter points in Forefront)
Performance Levels:

- Both accurate
- Not yet (<2 points)

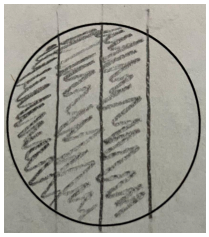
Answer Key:



4 regions should be approximately equal in size, and three of the sections should be shaded.

Readiness expectation: This task is looking for 2 elements:

- Students should recognize that the shapes need to be partitioned into 4 equal parts
- Three of those sections need to be shaded to represent ¾.



If the student has split the circle using parallel lines, consider it incorrect. Conceptually, the student might know that the sizes of the partitions need to be identical (or they might not). Even if the student has indeed intended to make fair shares, this should be considered an important point for clarification, since it is nearly impossible to slice a circle with parallel lines and arrive at equal-sized regions.

It should be noted that this problem suggests a very basic understanding of fractions, and while students should be able to readily complete this task, being able to complete this task is not a strong indicator of fractional reasoning. This is because for some students this problem has become so rote, that they simply know to cut into 4 and shade 3, essentially working through the process with only whole number thinking.

Part B Show 4/6 of a rectangle FR.3.Shapes (Shapes)

Performance Levels: Enter 1 or 0 into Forefront

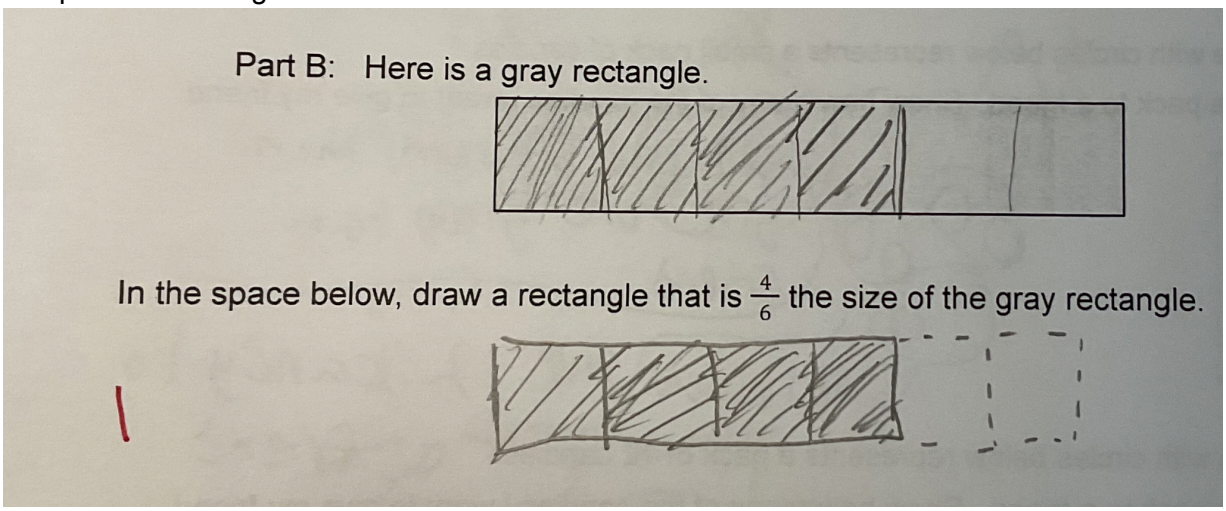
- Shows reasoning (1) - student accurately draws a rectangle that is 4/6 the size of the gray rectangle and demonstrates partitioning and iterating.
- Not yet (0)

Scoring Guide:
Student demonstrates an ability to reason that the gray rectangle needs to be divided into 6 equal parts, and then a rectangle the size of 4 of those parts should be drawn in the space below. If a student has hand drawn a rectangle approximately the correct size without any indication of how that was done, encourage the student to explain their answer.

As you circulate the room during the administration of the task, look for students who are attempting to partition the rectangle into sixths using their fingers, or pencils, etc. (see note about not using rulers below).

Students should apply partitioning (breaking the gray rectangle into 6 equal sized pieces), and then iterating (repeating the sixths) to create the rectangle which is approximately accurate in size.

What if they show ⅔? 4/6 was chosen as the fraction for this question, for its potential for revealing student understandings of equivalence. If a student draws a rectangle that shows that it is ⅔ the size of the gray rectangle, take note.
Sample of reasoning:



In this example, the student has shown how they partitioned the gray rectangle into 6 equal parts and reproduced the rectangle below. Notice how the student also shows the two-sixths that have been removed

Readiness expectation: This problem takes two steps toward complexity to see whether students have fully understood the idea of partitioning and iterating. By using 6 for the denominator it forces the student to think carefully about where those partitions need to happen, and perhaps adjust some. The second complexifying aspect of the question is asking the student to reproduce the fractional piece below. This is a step toward students understanding fractional parts of a whole while not destroying the original whole. This is called “disembedding” (Hackenberg, Norton & Write (2016)

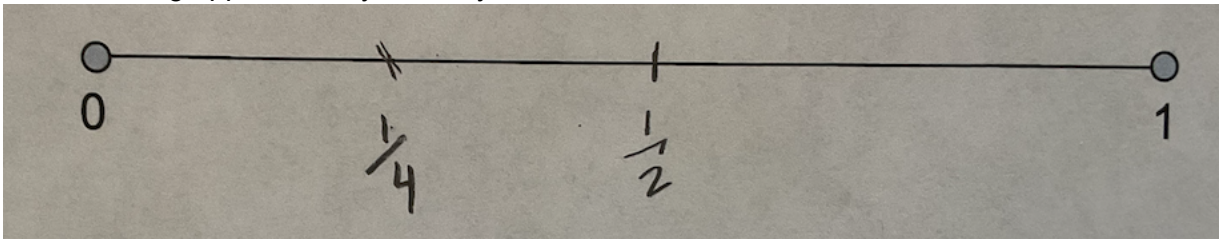
3. <u>Part A: Half and Fourth on Number Line</u> FR.3.NL (Number Lines)

Points possible: 2
Performance Levels:

- Shows reasoning: Both correct
- Not yet: Not both correct (<2 points)

Scoring Guide:
Both fractions should be correctly placed with ½ being approximately half way between 0 and

1 and ¼ being approximately half way between 0 and ½.



Readiness expectation: This task represents a starting point for fractions on number lines. Students who are unable to accurately place ½ and ¼ on a number line should be provided with small group instruction.
For students who finish quickly, and for more information, consider asking them where the number ⅓, and perhaps ⅕, should go on the line. While this is not scored on this rubric, these questions can reveal more about students' understanding of unit fractions on number lines and thus provide important formative information.

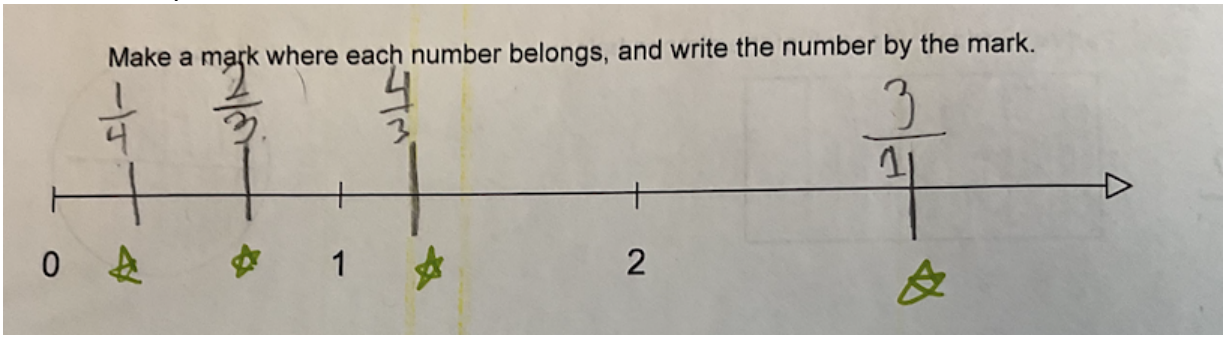
Part B: Fractions on Numbers Number Line FR.3.NL (Number Lines)

Points possible: 4
Performance Levels:

- Shows reasoning (4 points)
- Not yet (<4 points)

Scoring Guide:
Give one point for each fraction if:

- ¼ is left of center between 0 and 1.
- ⅔ is right of center between 0 and 1.
- 4/3 is left of center between 1 and 2.
- 3/1 is placed where the number 3 would be.



Readiness expectation:
Students should be able to represent fractions on number lines that extend beyond 1. The importance of the line extending beyond 1 is that it forces the student to first consider the whole before partitioning it. The ability to accurately place fractions on lines demonstrates their ability to reason about the magnitude of the fraction relative to the unit.

To place the numbers accurately, watch for students who are measuring with their fingers or pencils to divide the units into fractions. Students may make additional marks on the line to support their thinking. This is good evidence of their understanding that they need to break

the whole numbers into the appropriate number of pieces and then, if necessary, iterate those to correctly place the fractions on the line.

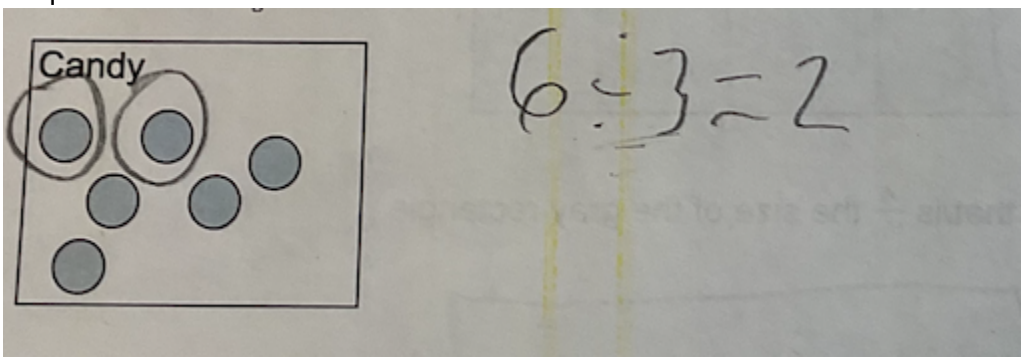
This question can reveal students who are still struggling with thinking about fractions as numbers, distinct from whole numbers. Many students imagine ⅔, for example, belongs somewhere between 2 and 3.

Note: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long.

4. Part 1: One Third of a Set of Six FR.3.Sets (Sets)

Performance Levels: Enter 1 or 0 in Forefront
- Shows reasoning (and correctly answers 2.) (1)
- Not yet (0)

Scoring Guidance:
Students need to recognize that in order to find ⅓ of a set, the set needs to be partitioned into 3 equal sized sets.



If a student simply answers two, and the thinking is not clear, as the student, “Can you tell me how you know?” Encourage students to write or draw an explanation of their answer.

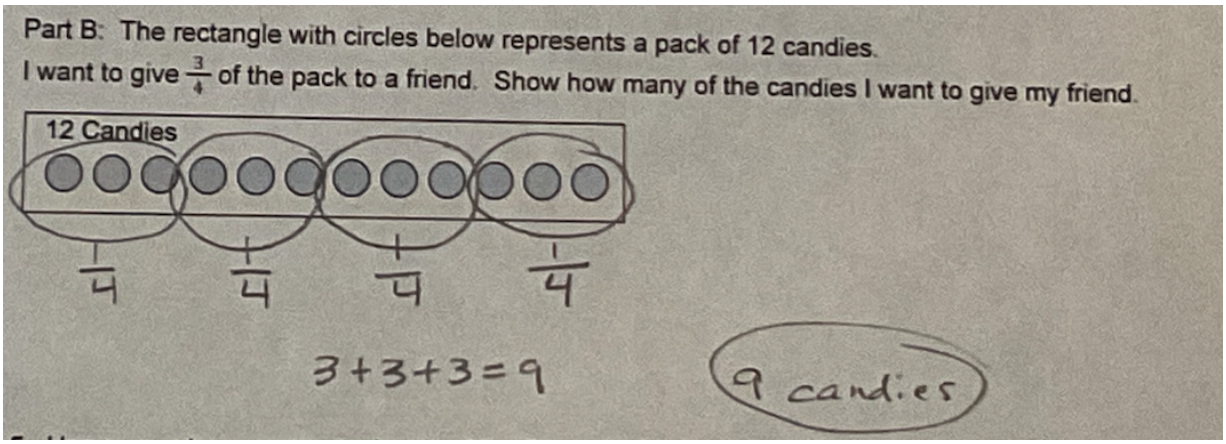
Readiness expectation:
While the ability to find a fraction of a set is not clearly defined in most state standards, the idea of forming equal groups is associated directly with ideas of whole number division. Students should be able to demonstrate that fractions of sets can be found by partitioning the larger set into smaller groups of equal size. Many students will relate this to the work with whole number multiplication and division that is central to the work of 3rd grade, and that is good.

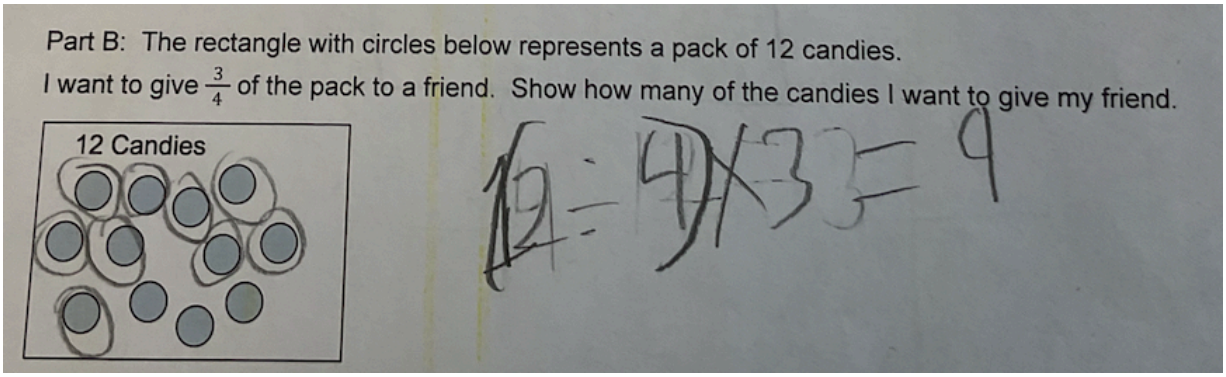
4. Part 2: ¾ of a Set of 12 Fr.3.Sets (Sets)

Performance Levels: Enter 1 or 0 in Forefront
- Shows reasoning (correctly answers 9 candies) (1)
- Not Yet (0)

Scoring Guidance:
Reasoning on this task is visible when the student has clearly found the unit fraction (¼) is

equivalent to 3 candies. Further, these students show that three 1/4s are represented by 9 candies.





5. Comparisons FR.3.MCE (Magnitude, Comparisons & Equivalence)

Points possible: 3, one for each correct comparison with explanations (enter points in Forefront).
Performance Levels:

- Demonstrate the use of reasoning about size to compare basic fractions (3 points)
- Not yet (<3 points)

Scoring Guide:

- It is not enough to simply put a symbol. If a student has only answered with a symbol, ask them to explain their answer with pictures, numberlines, words, or numbers.
- Students should explain their reasoning about the size of the fractions in order to compare them. Score their reasoning. If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<,>,=) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol and give a point for showing reasoning about size.
- If the student is unable to reason about the size of the fractions, but instead applies some sort of procedure to correctly compare the fractions, ask the student if they have

another way to show how they know their answer is correct. For example, if a student says that they know that ⅓ is greater than ¼ because the denominator is smaller, ask them to make a drawing to show how that works.
- If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, or other means to explain their reasoning about the size of the fractions, DO NOT give a point, even if the answer is correct.

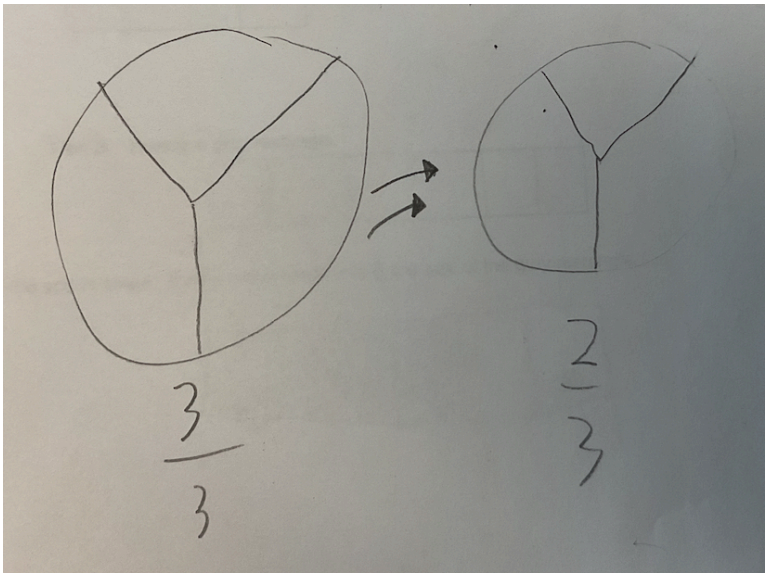
6. 1 - ⅓ of a Circle FR.3.C (Computation)

Performance Levels (enter 1 or 0 in Forefront)
- (1) Student shows the ability to reason about the circle and the missing piece to know that ⅔ of the whole circle is what remains.
- Not yet (0)

Scoring Guide:
- This question requires that the student answers ⅔ , and draws an image that shows approximately ⅔ of a circle. What we are looking for here is the ability to visualize the fractional amount and to tie that to the symbol.

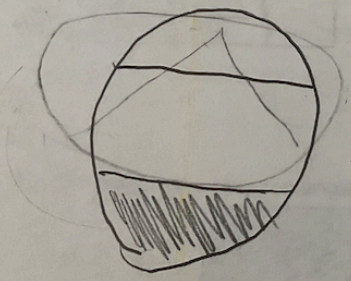
A correct answer shows the number ⅔ as the amount remaining and a drawing that approximates ⅓ of a circle being removed from a whole circle.

Sample:



The sample below should also be scored as “shows reasoning.” (1). Although the partitions are not even in the drawing, and do not reflect exactly what was in the video, as it relates to the problem solving and arriving at the correct answer, the student has demonstrated the reasoning that this question is assessing.

6. Video A circle is divided into 3 equal parts. One part is cut off and removed. How much of the circle remains? Write a fraction and draw to show your answer.



$\frac{2}{3}$

$\frac{2}{3}$ still remain because your just taking one away.

Instructions for the Administration of
The 4/5 Fractional Reasoning Screening Assessment

This assessment is intended to be administered to the whole class at once. The teacher will be asked to read a script in order to establish a pace for the assessment and keep the time demands minimal. There will be students who are unable to complete any of the tasks during the time provided, plan to provide the time at the end of the assessment or at a later time.

When should this assessment be used? This assessment has been designed to be used at one of the three times below.

Mid 4th Grade: Administer in the midst of 4th grade fractions instruction as a formative assessment.

End of 4th Grade: Administer the assessment after the conclusion of instruction related to fractions in 4th grade to identify areas for re-instruction, additional practice, and small group work.

Early 5th Grade: Use this assessment before or early in the instruction of fractions and proportions in 5th grade to gather formative information to inform topics and students for pre-teaching before formal instruction with fractions begins.

Time: Plan for **45 minutes** for the whole group administration and additional time for extended time for targeted students. With some groups the administration of the assessment might be done in as little as 30 minutes.

Preparation:

- Photo copies of the assessment for all students
- Pencils with erasers (not pens) for all students
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the video: There is a video that accompanies this assessment for question 6. Have the video cued before you begin the assessment.

Video for #6

Assessment administration:
Please follow the script as written and have a clock or timer ready. This will help to keep the assessments more reliable. It will also ensure that the assessment remains efficient. Students should not be aware of the timing of the tasks. This is not a timed test.

Answer questions only as necessary to ensure students understand the task. Refrain from answering questions that would influence student thinking or provide them clues to how to solve the tasks.

Active Proctoring: Teachers are encouraged to move about the room and to interact appropriately and actively with the students.

- Where there are students who seem to not be engaging, it is appropriate to encourage them, and to re-read the question for them personally to ensure understanding.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning.
- When you see student work that leaves you uncertain about their reasoning, ask questions and encourage them to draw, use number lines, or demonstrate their thinking in other ways.

Observations: While systemic collection of observational data is not included in this assessment look-fors are called out in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring: Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the student to ensure the accuracy of the analysis.

Forefront users: Forefront has been configured to support data collection for Forefront clients. If you are interested, please reach out to support@forefront.education to request that the assessments be added to your district's resources.

Accommodations: Any special accommodations which would normally be given for individual students should also be given for this assessment.

Grade 4/5 Script: Fractional Reasoning Screener

Before you begin the assessment, prepare the video to be shown for Task 6.
https://drive.google.com/file/d/1BrdjCV06stnvV33B_njsLwAbpMC44z0r/view

Read these instructions completely to familiarize yourself with the instructions in preparation for the assessment passing out the papers. Teachers should read all of the bolded text.

Prepare a timer before you start, but be discreet with it. This is not a timed test, but the timings will help you to keep the students on a reasonable pace as they work through the assessment. The suggested timings are intended to be guidelines; teachers are encouraged to use their best judgment as they lead the students through the assessment. Once almost all of the students have completed a task, let students know that you are going to continue to the next task, but that they will be provided with time to return to unfinished tasks later.

Teacher: **Today we are going to solve some problems to help me understand how you understand fractions. The results of this assessment will not be part of your grade. This is only to help me understand how you think about fractions so that I will be able to teach you better.**

This is an opportunity for me to learn how you think about these kinds of questions, so it will be important for you to show your thinking on each of the tasks, with words, pictures, numberlines, and numbers.

It's important that I learn about your own ways of thinking about these problems, so please keep your eyes on your own papers. (Some teachers may want to spread students out a bit, and/or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please do not work ahead, since these questions are a little different than other assessments. The last question will include a video that we will watch together.

If at any time you have a question, please raise your hand. Are there any questions before we get started? (answer questions as necessary)

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

Task 1:

When students are ready, read the script. Pause after each of the numbers to give students the opportunity to write their numbers. Continue at a reasonable pace. You may say the numbers more than once if necessary.

Teacher: **For the first question I will read some numbers aloud. Write the numbers in each of the boxes.**

In box A write the number: one and one half.
In box B write the number: four sevenths.
In box C write the number: six thirds.

Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Task 2:
Teacher: **Find questions 2a on your paper. 2a says, "This rectangle represents a piece of chocolate. This piece is three fifths of a whole chocolate bar. In the space provided, draw a whole chocolate bar."**

Use lines, make marks, and show your thinking.
Clarify if necessary to ensure that the students understand the task.

Give the students 2-3 minutes to work on 2a. Circulate the room to monitor progress. Encourage students to show their thinking. Ask them to show how they know what they have drawn is the right size, and how they can prove it.

Teacher: **I know some of you are still working, but let's take a look at Question 2b. 2b has some squares. "Write a fraction for the shaded portion of each square below." Write your fractions below or beside the square. You may draw on the squares to help you.**

Teacher: **Let's look together at question 2c. Turn the page and you will see question 2c. It says, "The gray shaded area shows one third stick of butter. Below are some more sticks of butter. Shade to show five thirds sticks of butter."**

Allow 2-3 more minutes for questions 2a, 2b, and 2c (or continue along earlier if everyone has finished). When most of the class is ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Continue to task 3.

Task 3:
Teacher: **Turn the page to find question 3a. Question 3a says, "Place these numbers on the number line. Make marks and write the numbers to show the position of each number."**

Give the students 1-2 minutes to work on question 3a.

When most students are ready, say, "**Look at question 3b. It says, "What fractions go in the boxes?" Look at the boxes below the line. Write the fractions that should go in each box."**

Allow another 2-3 minutes for questions 3a and 3b (or continue along earlier if everyone has finished). When almost everyone is ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Task 4:
Teacher: **Number four. Number 4a says, "The three dots below represent one eighth of a package of candies. Draw five eighths of a package of candies in the box."**

Below you will see a line that says, "How many candies are in five eights of a package." Write the number of candies on the line.

After students have had a chance to answer, after 1 -3 minutes, say, **"Please turn the page and look at question 4b. Question 4b says, "The dots below show three fourths of a pack of candies. How many candies are in a full pack of candies? Solve the problem, show you thinking and write the number of candies in the full pack on the line.**

Allow another 3 minutes for questions 4a and 4b (or continue along earlier if everyone has finished). When ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Task 5:
Teacher: **Questions 5a and 5b. Question 5a says, "Jan says six tenths is bigger than three fifths. Do you agree or disagree with Jan? Explain your answer. Include pictures or number lines in your explanation."**

Give 2-3 minutes for students to start working on 5a.

Let's take a look at question 5b. Question 5b says, "Compare the following fractions using the greater than, equal to, or less than symbols. Explain your reasoning for each, using pictures, words, lines, and/or numbers."

Work on questions 5a and 5b now.

Allow 5 minutes for questions 5a and 5b (or continue along earlier if everyone has finished). After 5 minutes, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

As students work on these tasks circulate through the room.

- When students provide only numeric justifications for their reasoning, ask them if they have other ways that they can prove it.
- Where students use only drawings, ask if they have other ways they can show their reasoning. It is appropriate for students to also use words to explain their thinking.

Task 6
Teacher: **Number 6. For this question I will read it, and then we will watch a short video that goes with the question. I will tell you when to start after we watch the video.**

A rope is 2 ft. long. $\frac{2}{3}$ of a ft. is cut off. How long is the remaining rope? Play the video.

Solve the problem. At the bottom, write an equation that matches your solution.

Give the students a few minutes to solve the rope problem. Circulate the room and encourage them to show their thinking and remind students to write an equation to match the problem.

Note: If you see a student who answers problem 6 with inches (1 ft. 4 in.) ask them if they can also write their answer as a fraction.

Collect papers from students as they finish. As you do, check to see that their answers are complete. If there are incomplete papers, encourage students to finish those problems up, or to elaborate on their answers as necessary. Provide extended time to those who need it, or if necessary, separate those papers to allow those students to complete the work at a later time.

4 / 5 Fractional Reasoning - Show what you know.

Name _____ Date _____

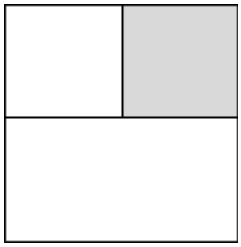
1. Write the numbers in each of the boxes as your teacher reads them.

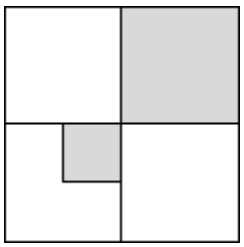
A	B	C

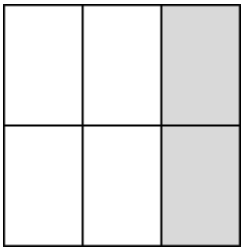
2a. This rectangle represents a piece of chocolate. This piece is $\frac{3}{5}$ of the whole chocolate bar.

In the space below, draw the whole chocolate bar.

2b. Write a fraction for the shaded portion of each square below.







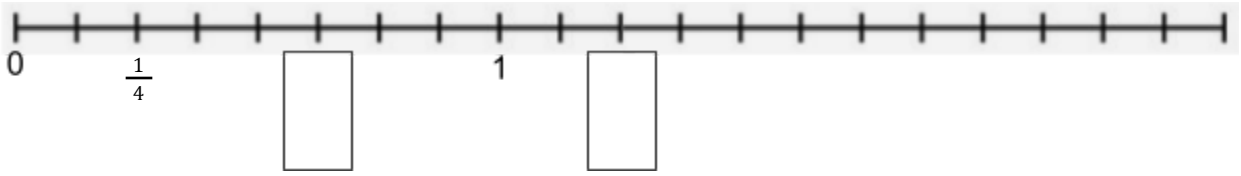
2c. The gray shaded area shows $\frac{1}{3}$ of a stick of butter.

Below are some more sticks of butter. Shade to show $\frac{5}{3}$ sticks of butter.

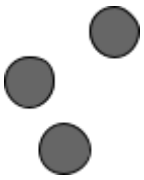
3a. Place these numbers on the number line. Make marks and write the numbers to show the position of each number.

$1\frac{1}{2}$ $\frac{2}{3}$

3b. What fractions go in the boxes?



4a. The 3 dots below show $\frac{1}{8}$ of a pack of candies. Draw $\frac{5}{8}$ of a pack of candies in the box.



This is $\frac{1}{8}$ of a package.

Draw $\frac{5}{8}$ of a package here.

How many candies are in $\frac{5}{8}$ of a package? _____

4b. The dots below show $\frac{3}{4}$ of a pack of candies. How many candies are in a full pack of candies?

This is $\frac{3}{4}$ of a pack.

How many candies are in a full pack? _____

5a. Jan says $\frac{6}{10}$ is equal to $\frac{3}{5}$. Do you agree or disagree with Jan? Explain your answer. Include pictures or number lines in your explanation.

5b. Compare the following fractions using >, =, <. Explain your reasoning for each, using pictures, words, lines, and/or numbers.

$\frac{2}{5}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{6}{8}$ $\frac{8}{7}$ $\frac{7}{8}$

6. A rope is 2 ft. long. $\frac{2}{3}$ of a ft. is cut off. How long is the remaining rope?

Solve the problem below. Use words, pictures, lines, or numbers to explain your thinking.

Write an equation to match your solution. _____

Fractional Reasoning Screener
4/5 Scoring Guide

Forefront data entry: Enter results as instructed in each of the questions.

Standards alignments: At this time, these questions are simply aligned with the 3rd grade domain of fractions (e.g. 4.NF). Specific alignments for different states will be done and configured in Forefront over time.

Fractional Reasoning Lens: Alignments to the Fractional Reasoning Lens are listed with each of the tasks.

Observations: For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These should be taken into consideration during the scoring.

1. Writing Fractions from Dictation — FR.4.WS (Words and Symbols)

Possible points: 3, one point for each correctly written fraction (enter points in Forefront). Performance Levels: ● All accurate (3 points) ● Not yet (< 3 points)
Answer Key: $A.\ 1\frac{1}{2}$ $B.\ \frac{4}{7}$ $C.\ \frac{6}{3}$ Give one point for each correctly written fraction.
Clarifications: Credit should be given if the number is legible and accurate. While students should be encouraged to write the numerals vertically, give points if they have written them like this 1 ½.
Readiness expectation: Students should be able to write fractions and mixed numbers.

2a. Given ⅗ of a rectangle, show the whole rectangle — FR.4.Shapes (Shapes)

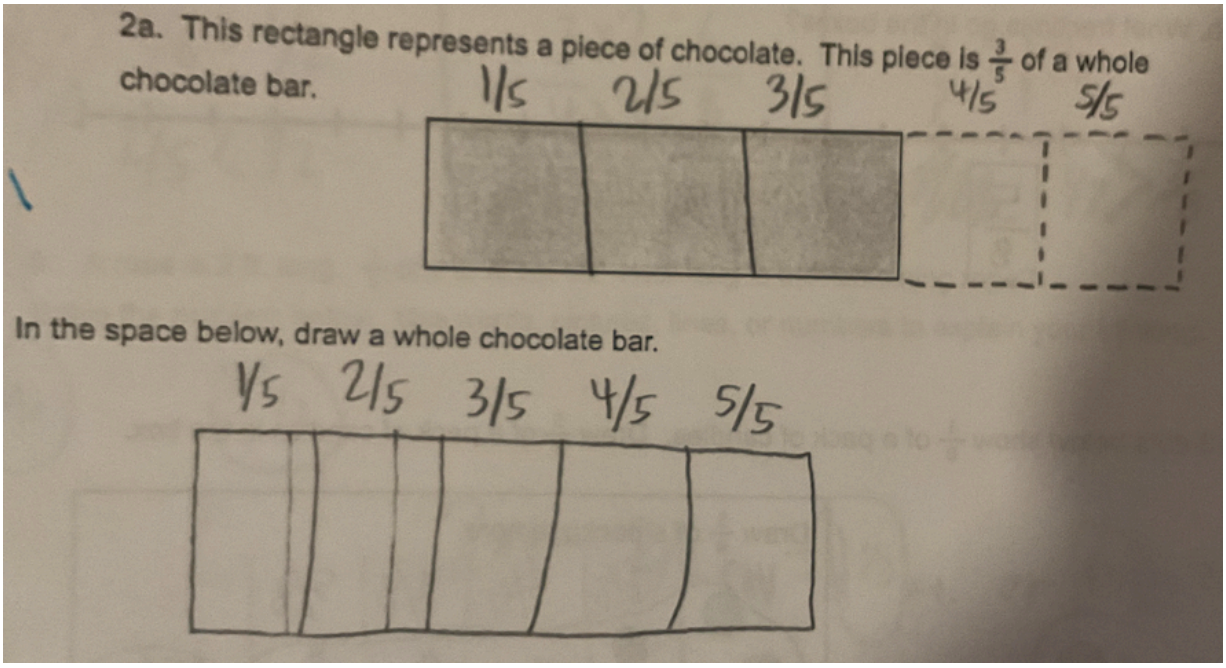
Performance Levels: Enter 1 or 0 into Forefront ● Shows reasoning (1) - student accurately draws a rectangle that is two fifths larger than the gray rectangle and demonstrates partitioning and iterating. ● Not yet (0)
Scoring Guide: Students will reason through this in a variety of ways, but the most common to look for is when students have reasoned that they need to find the size of the unit fraction by dividing the "chocolate bar" into 3 pieces. To create the whole chocolate bar, the unit then needs to be

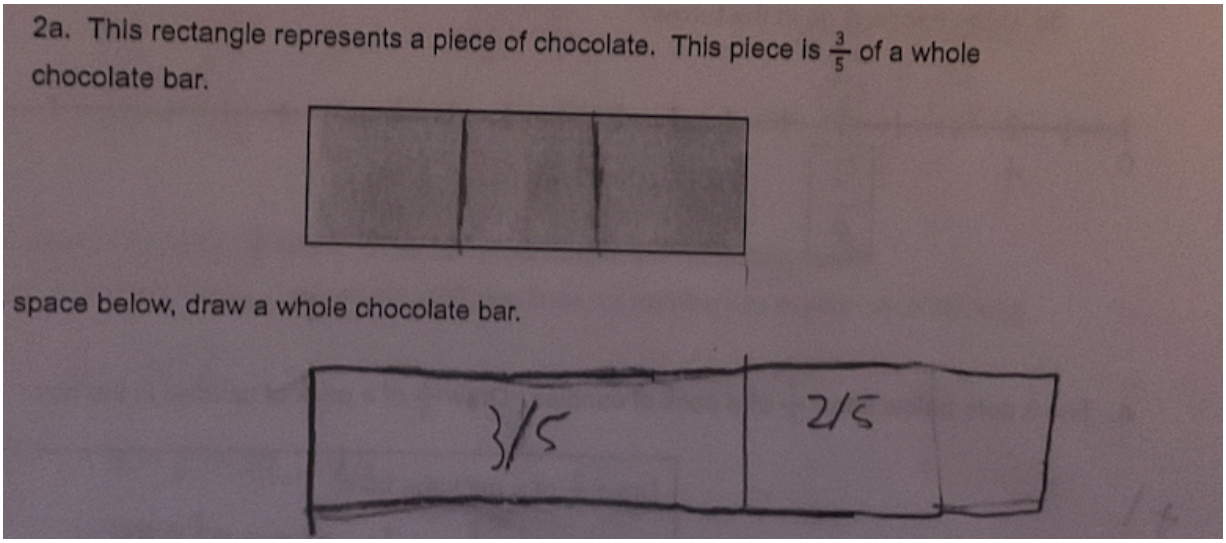
repeated (iterated) 5 times.

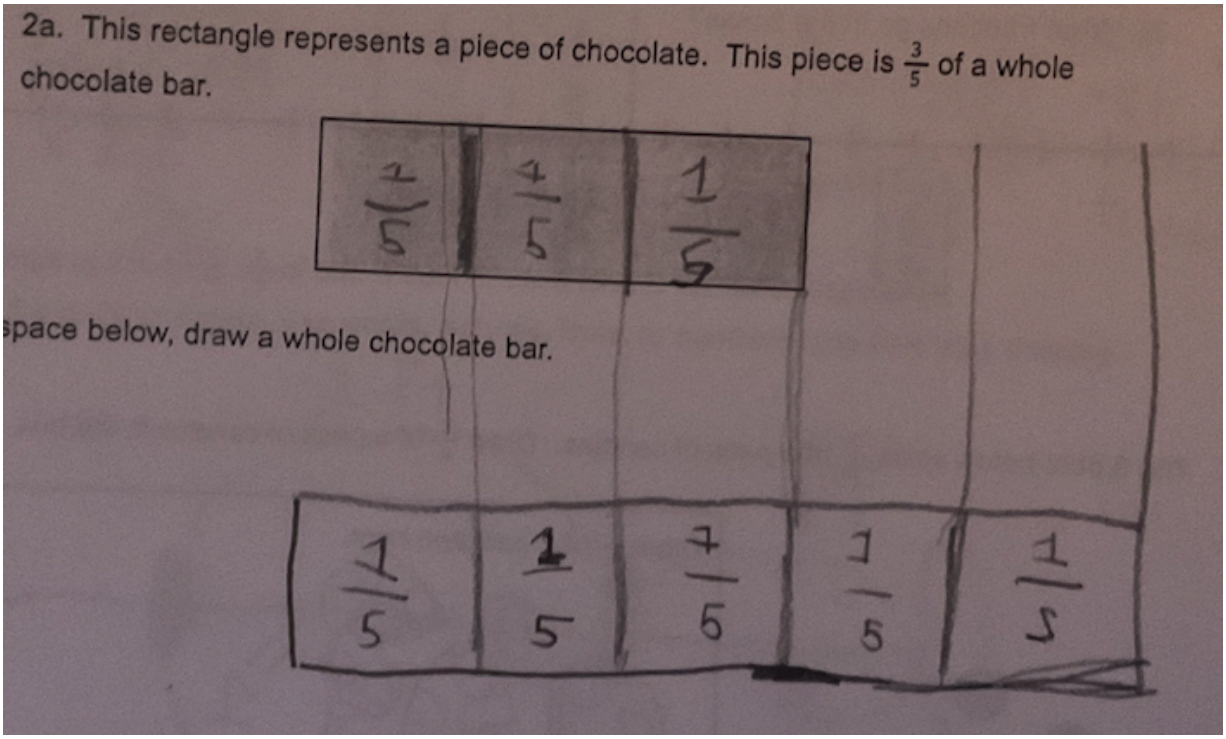
As you circulate the room during the administration of the task, look for students who are attempting to partition the rectangle into 3 equal parts, using their fingers, or pencils, etc. (see note about not using rulers below).

Precision is not of primary importance. Look for evidence of reasoning.

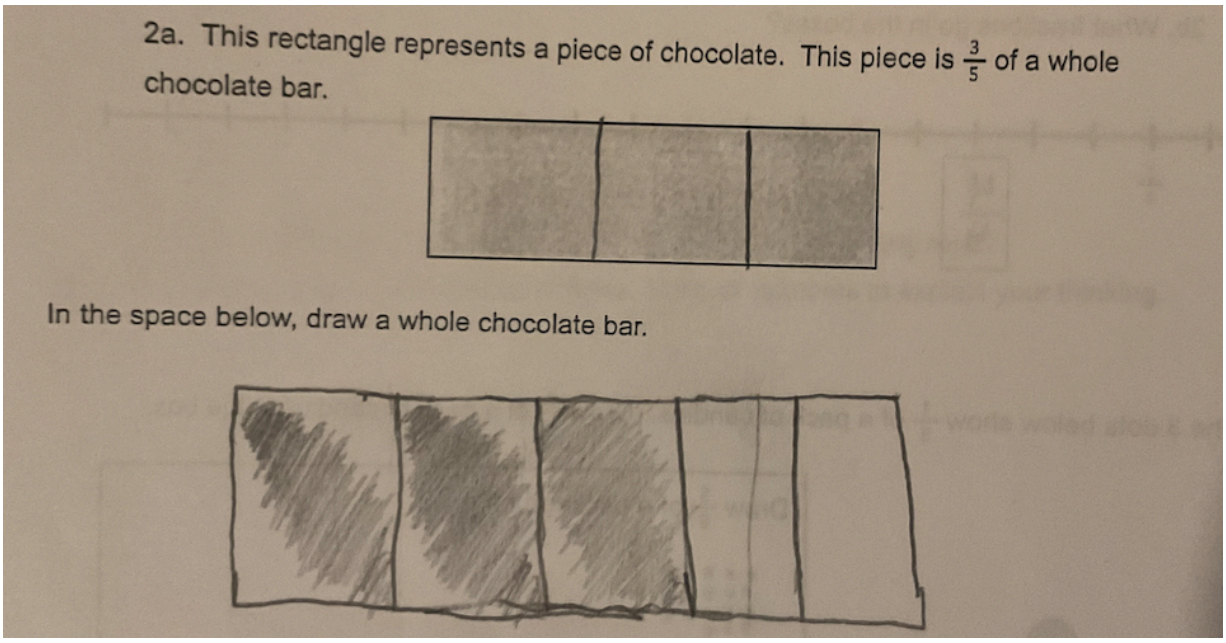
Work Samples:
These samples represent clear reasoning. The original chocolate bar has been partitioned into 3 sections to find the unit and then that unit has been iterated 5 times to represent the full chocolate bar.



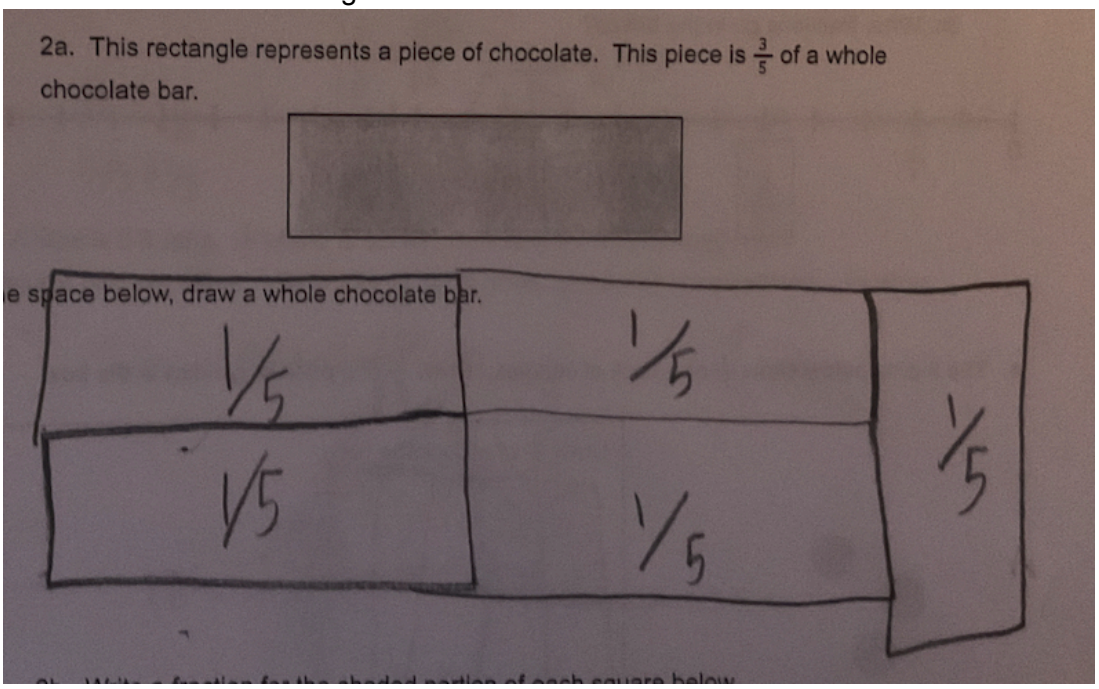




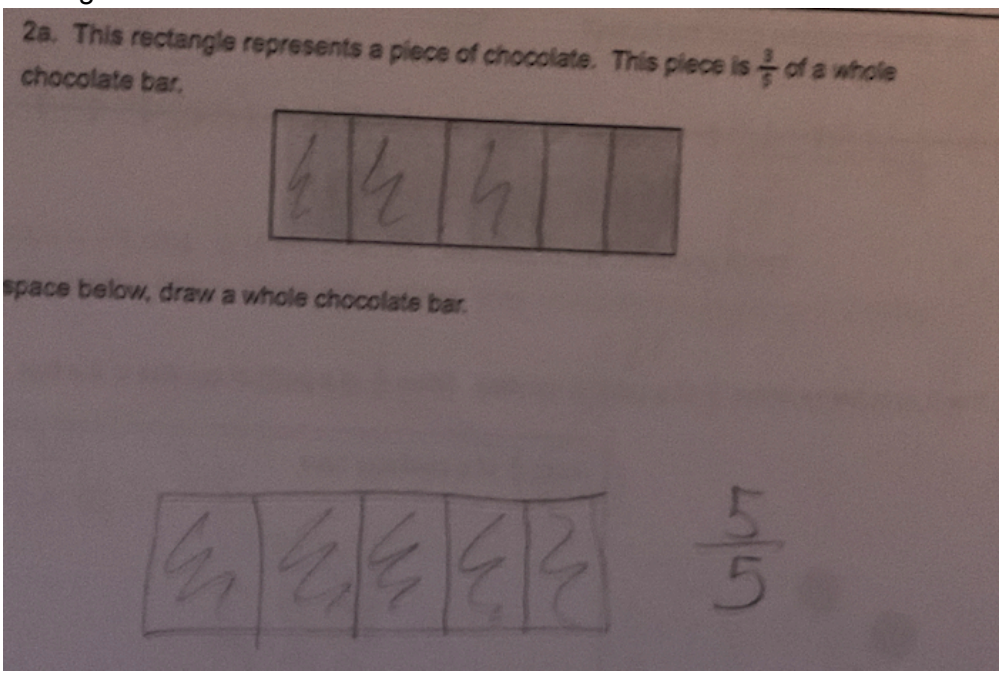
While less clear, the student here has found the unit fraction and iterated it. This should be scored as "shows reasoning."



Common misunderstandings: These solutions should be scored as "Not Yet."



Notice on this sample that the student has divided the original chocolate bar into 5 equal parts, thus their unit size is inaccurate, even if they have indicated that the whole bar would be longer.



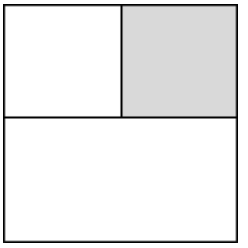
Commentary: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long. We can see that a student applies partitioning and iterating in their response without the need for precise measurements.

2b. Area of Squares: Reasoning about Equivalence FR.4.Shapes (Shapes)

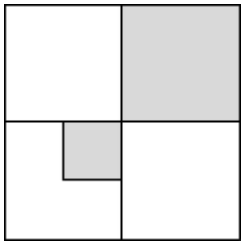
Points possible: 3 points (enter points in Forefront)
Performance levels:

- All correct - Meets expectations
- 1 or 2 correct - Approaching expectations
- None correct - Not yet

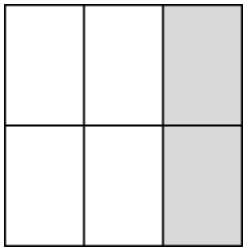
Scoring Guide:
Give one point for each correct answer.



$\frac{1}{4}$

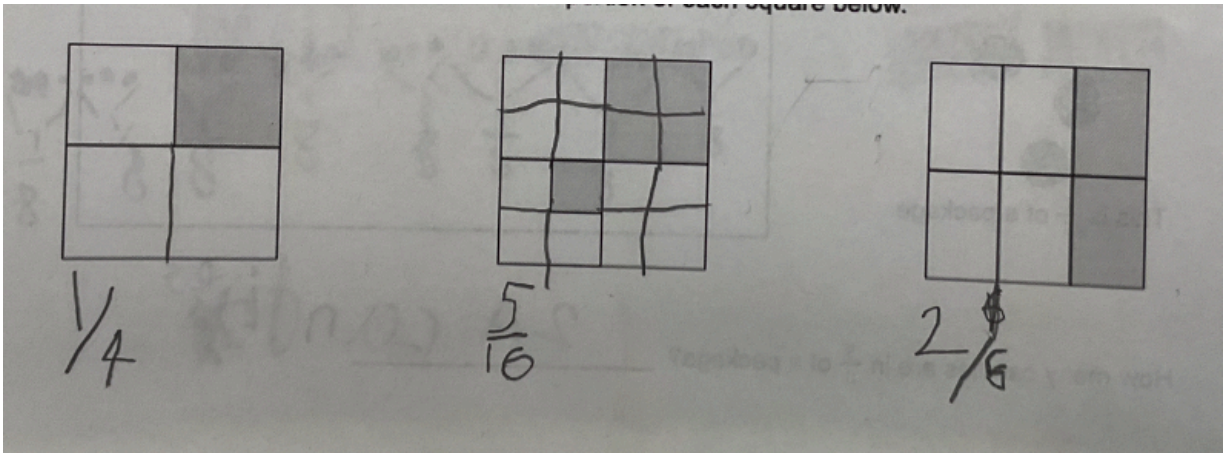


$\frac{5}{16}$

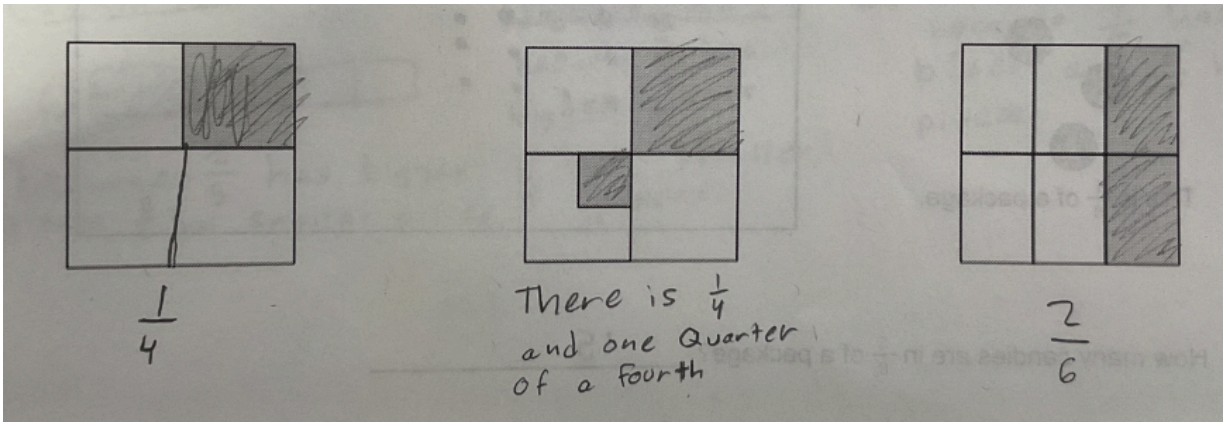


$\frac{2}{3} or \frac{2}{6}$

Accept equivalent fractions for any of the answers. However, if a student gives an unusual answer like 2/8 instead of ¼, then ask the student to explain.



While the second answer in the image below should be scored as incorrect, notice the brilliance of the answer this student has given. “There is 1/4 and one quarter of a fourth.”



Readiness expectation:
Students know that fractions of geometric shapes need equal sized pieces. Students are able to subdivide shapes to create and write fractions to match.

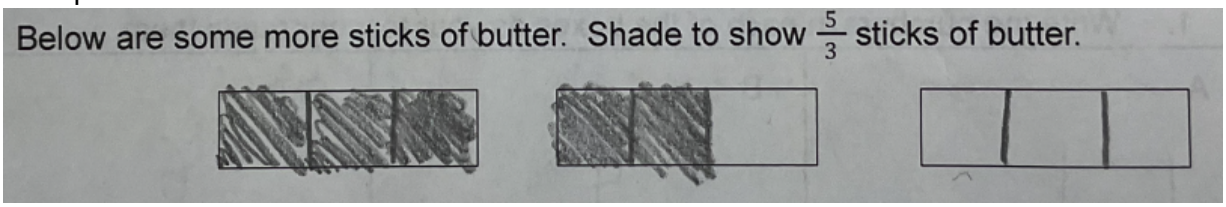
This question also helps to reveal those students who are able to readily recognize and generate equivalent fractions.

Proctoring and observations: As you walk around the room, if you see students who finish early ask if they can write equivalent fractions for each of the shapes (especially the last one).

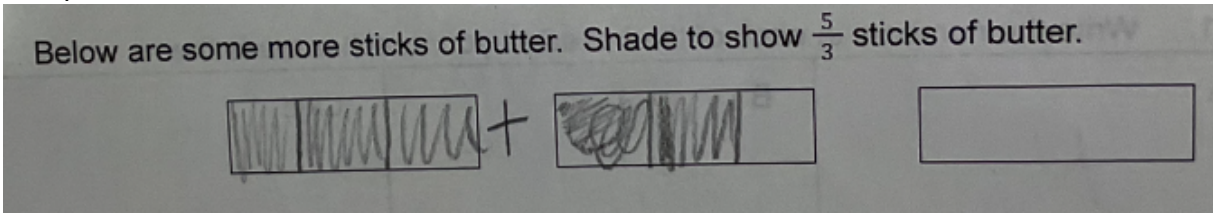
2c. 5/3 sticks of butter

Points Possible: Enter 1 or 0 in Forefront
Meets Expectations: 1
Not Yet: 0

Scoring Guide: Students should have shaded one full rectangle and 2 more thirds, or any other combination to make 5 thirds.
Example 1



Example 2



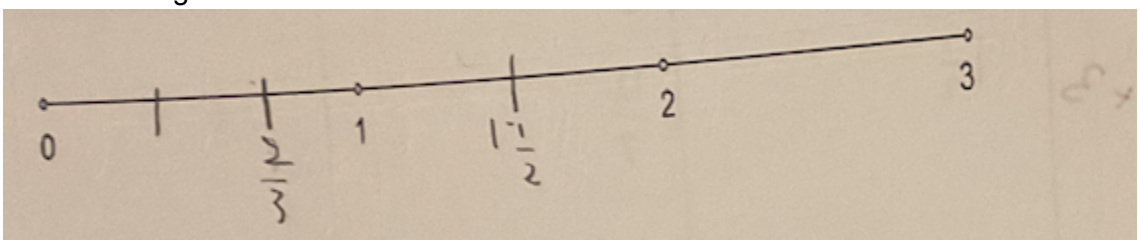
Performance Expectation: Students should know that the thirds need to be repeated 5 times to make a total of 5 thirds.

3a. Numbers on Number Line FR.4.NL (Number Lines)

Points possible: 2 points (Enter points in Forefront)
Performance Levels:
Meets Expectations (2 points): Student demonstrates the ability to represent and locate fractions on number lines.
Partially (1 point) Student has placed one of the numbers correctly on the line.
Not yet (0 points): Student is beginning to learn how to represent and locate fractions on number lines.

Scoring Guide:
Give one point for each fraction if:

- 1 ½ is in the middle between 1 and 2.
- ⅔ is right of center between 1 and 2.



Readiness Expectation: Students should be able to represent fractions on number lines that extend beyond 1. The importance of the line extending beyond 1 is that it forces the student to first consider the whole, and then partition it. The ability to accurately place fractions on lines demonstrates their ability to reason about the magnitude of the fraction relative to the unit.

Watch for students who are measuring with their fingers or pencils to divide the units into fractions. Students may make additional marks on the line to support their thinking. This is good evidence of their understanding that they need to break the whole numbers into the appropriate number of pieces and then, if necessary, iterate those to correctly place the fractions on the line.

Commentary: Although problem 3a could be solved with a high degree of precision using a ruler, it would simply take too long.

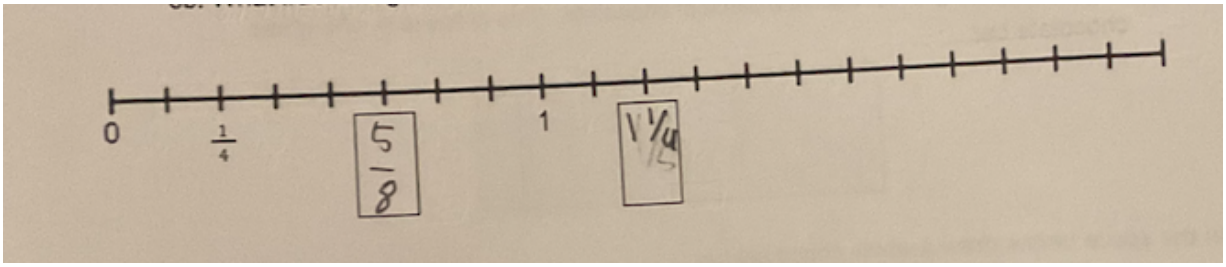
3a. Numbers on Number Line FR.4.NL (Number Lines)

Points possible: 2 points (Enter points in Forefront)
Performance Levels:
Meets Expectations (2 points): Student demonstrates the ability to represent and locate fractions on number lines.
Partially Meets (1 point) Student is developing the ability to represent and locate fractions on number lines.
Not yet (<2 points): Student is beginning to learn how to represent and locate fractions on

number lines.

Scoring Guide:
Give one point for each fraction if:

- ⅝ is in the box on the left.
- $1\frac{1}{4}\ or\ 1\frac{2}{8}\ or\ \frac{5}{4}\ or\ \frac{10}{8}$ is in the box on the right.



Readiness expectation:
This number line gets at two important ideas. The first is equal partitions. Students should recognize that there are 8 partitions between 0 and 1, and thus the first box is located at ⅝. The second idea that this problem seeks to elicit is the idea of equivalent fractions. Students entering 5th grade should be able to use number lines to recognize and express equivalent fractions.

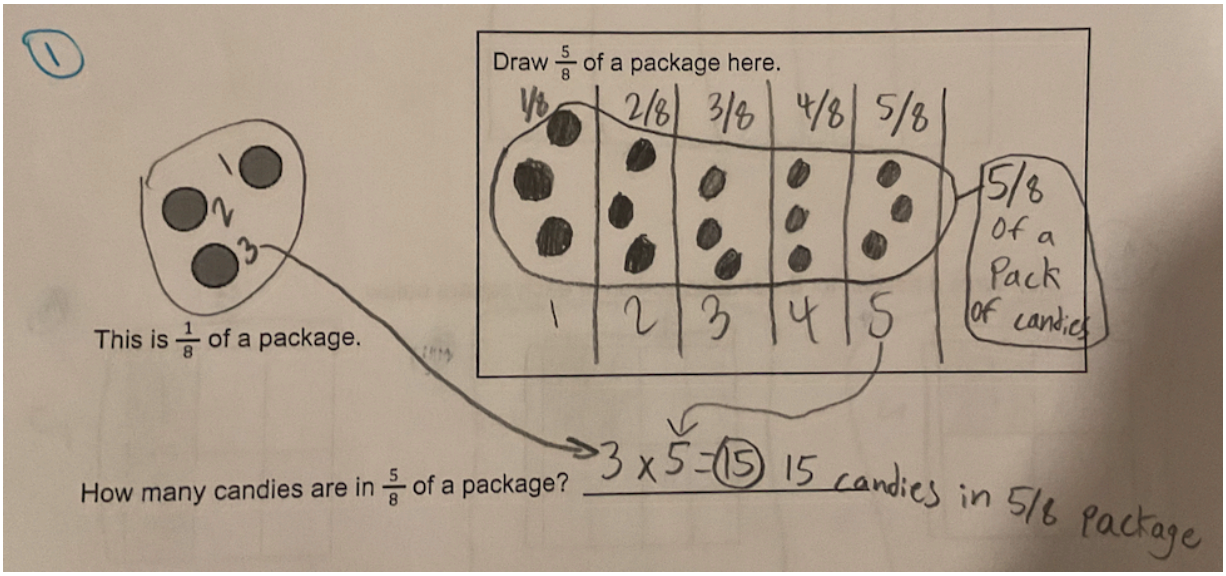
For students who finish quickly, consider asking if there are other possible fractions that they could put in the box on the right.

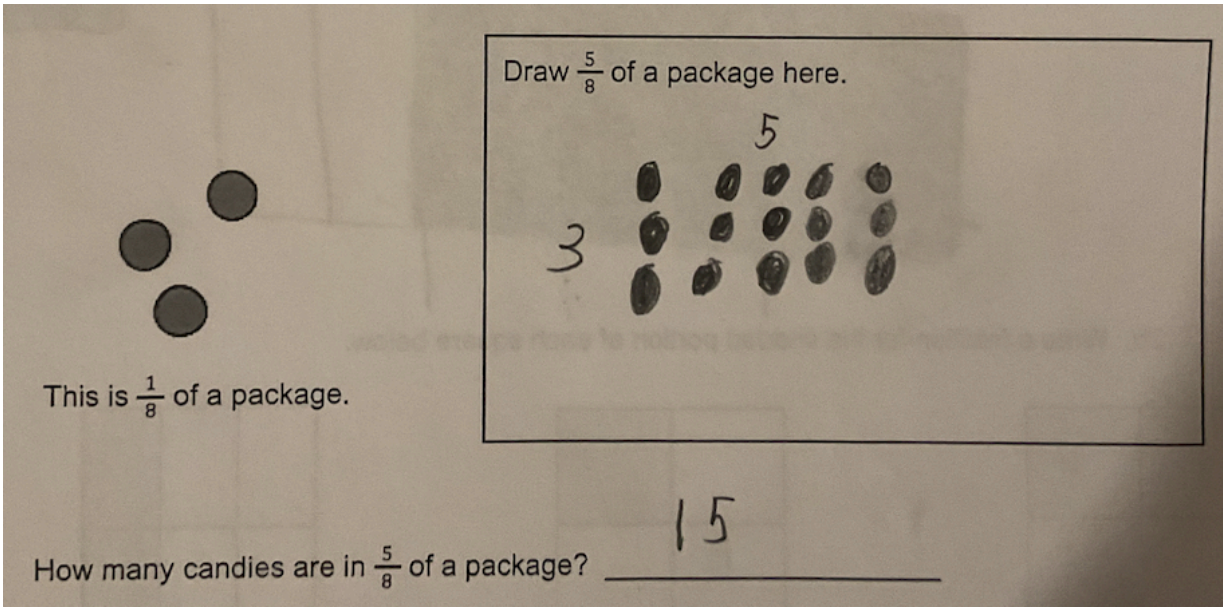
4a. 3 is one eighth of a pack. How many in ⅝ pack. FR.4.Sets (Sets)

Performance Levels: Enter 1 or 0 in Forefront

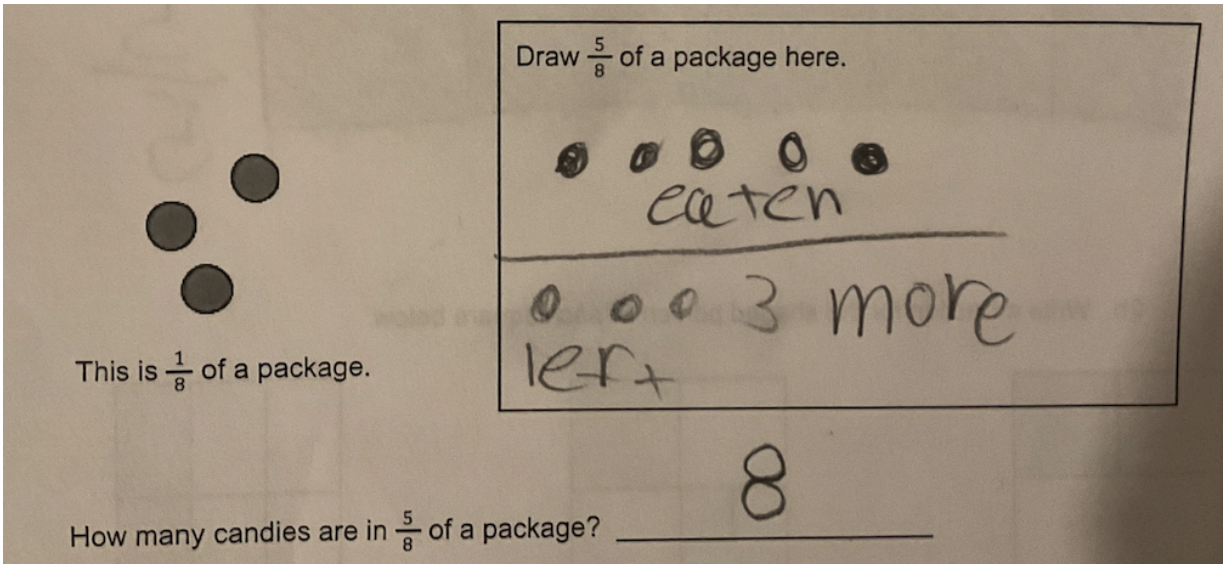
- Meets Expectations: Applies understanding of fractions to sets. Correctly answers 15. (1)
- Not yet (0)

Scoring Guidance:
Students should show 15 in the drawing. Students may arrange the dots in an array. Student answers 15.





This example shows the student does not yet understand how the unit of 3 candies needs to be iterated 5 times to show ⅝. Notice that the student is seeing this as a subtractive problem.



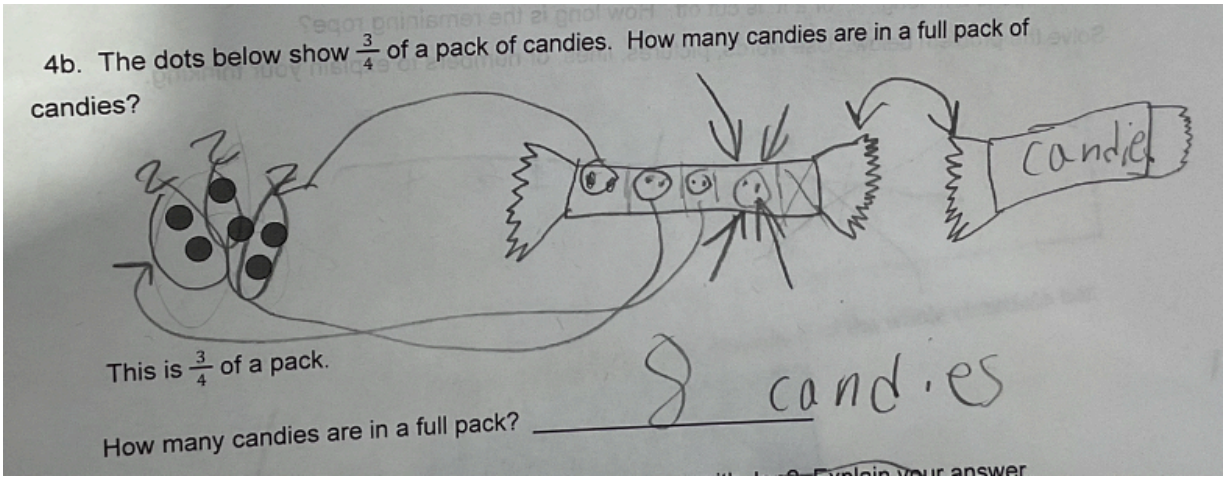
Readiness expectation:
This problem is about the ability to see a set as a unit fraction and to iterate that set to form a proper fraction. Students should be able to see that a set can represent a single unit that can be iterated.
This problem presents an interesting crossover of fractional and whole number thinking. Students will often see the need to repeat the set of 3 five times, and then will calculate the answer using whole number thinking.

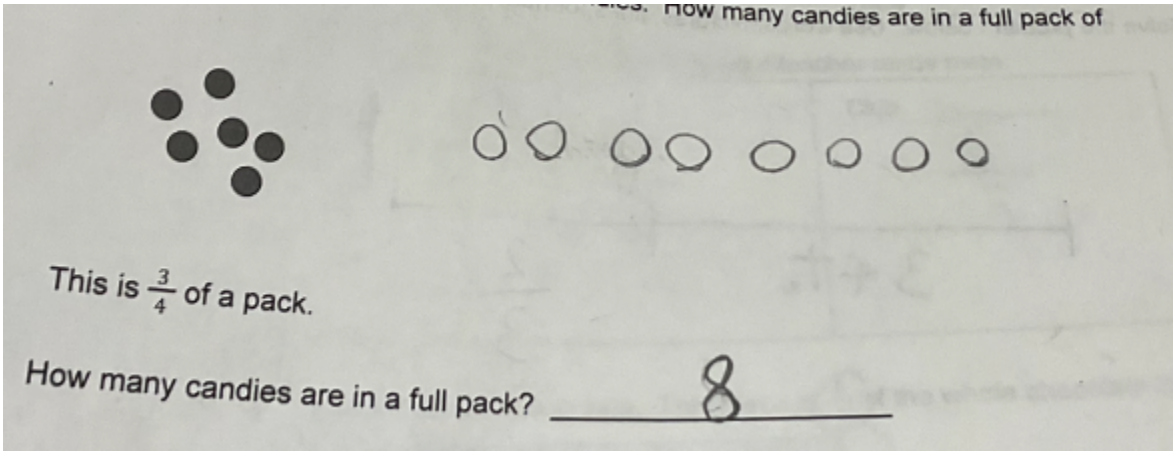
4b. <u>From a proper fraction, identify the whole</u> FR.4.Sets (Sets)

Performance Levels: Enter 1 or 0 in Forefront

- Meets Expectations: Applies understanding of fractions to sets to find the whole. Correctly answers 8. (1)
- Not yet (0)

Scoring Guidance:
Look for students to identify the unit of 2 candies and to add those two candies to the 6 that have been presented to see that a full pack would have 8 candies.





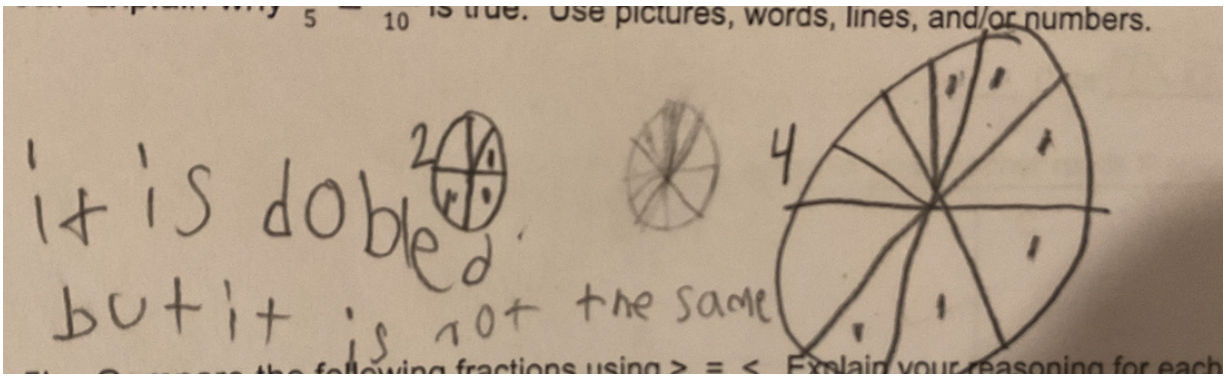
5a. Explain equivalence FR.4.MCE (Magnitude, Comparisons & Equivalence)

Performance Levels: Enter 0 or 1 in Forefront
Meets Expectations (1): Student demonstrates the ability to represent equivalent fractions
Not Yet (0): Student does not demonstrate an ability to represent an understanding of equivalent fractions

Scoring Guide:

- Students who meet expectations demonstrate the ability to communicate the meaning of equivalent fractions using shapes, number lines, words, and/or numbers. If a student has only provided an equation (e.g. $\frac{3}{5} * \frac{2}{2} = \frac{6}{10}$) ask if they can use a drawing or number line to explain why that is true.
- Look for students to show comparisons that show two equal sized wholes.

Look also for these kinds of common misunderstandings: Notice how the student creates two fraction circles, one which is twice the size of the other.



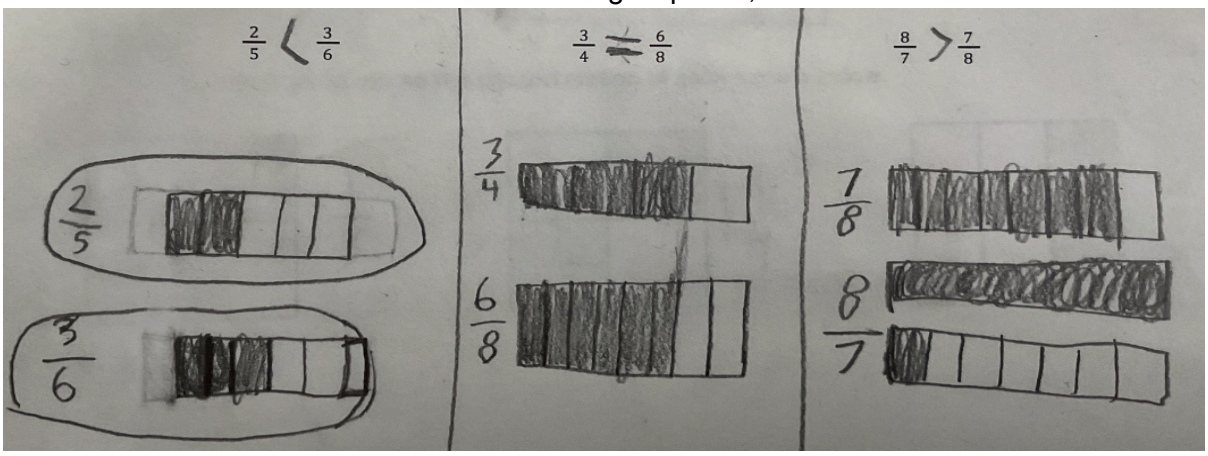
5b. Compare FR.3.MCE (Magnitude, Comparisons & Equivalence)

Points possible: 3 (Enter points in Forefront).
Performance Levels:

- Demonstrates the use of reasoning about size to compare basic fractions (3 points)
- Developing (2 points)
- Not yet (<2 points)

Scoring Guide:

- Give 1 point for each correct comparison with clear reasoning about the size and/or the equivalence of the fractions in order to compare them. Score the reasoning. If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<,>,=) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol, take note, and give a point for showing reasoning about size.
- If the student is unable to reason about the size of the fractions, but instead applies some sort of procedure to correctly compare the fractions as the student if they have another way to show how they know their answer is correct. For example, if a student says that they know that ⅓ is greater than ¼ because the denominator is smaller, ask them to make a drawing or use a number line to show how that works.
- If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, numbers, or other means to explain their reasoning about the size of the fractions DO NOT give points, even if the answer is correct.



6. 2 ft. - ⅔ ft. FR.3.C (Computation)

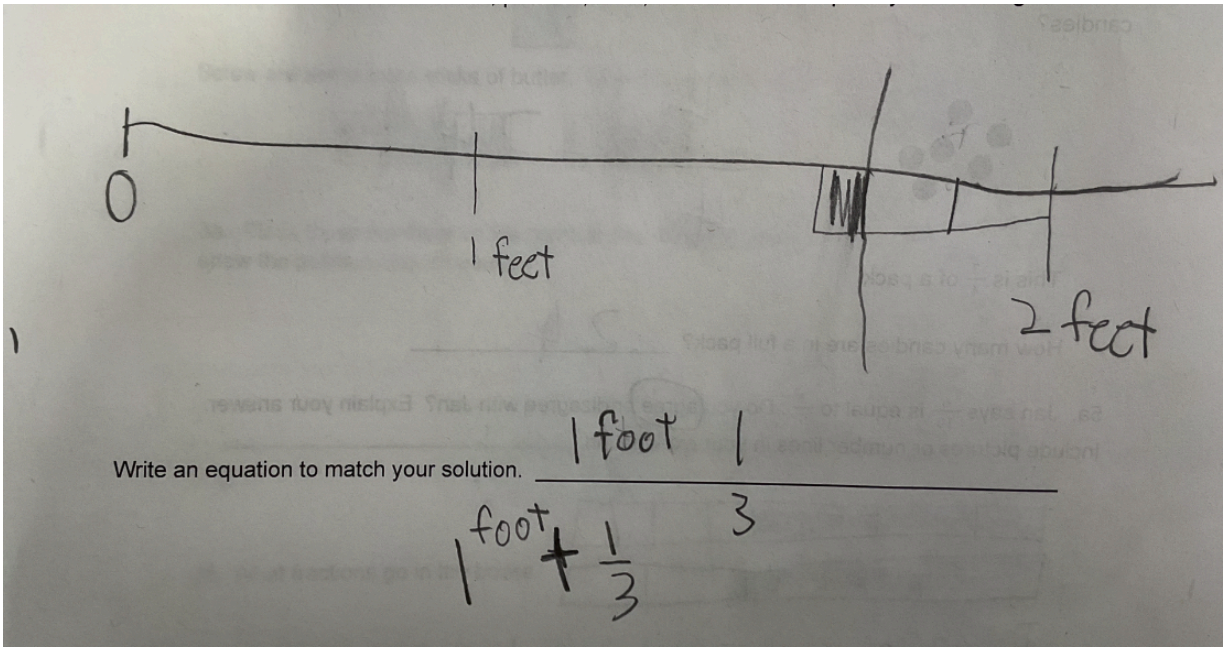
Points possible: 2 (Enter points in Forefront)
Performance Levels

- Meets Expectations (2 pts.): Student shows the ability to solve a simple subtraction problem with fractions and whole numbers in context, and write an equation that matches.
- Approaching Expectations (1 pt.): Student is either able to solve a problem or write an equation to match a problem, but not both.
- Not yet (0 pts.)

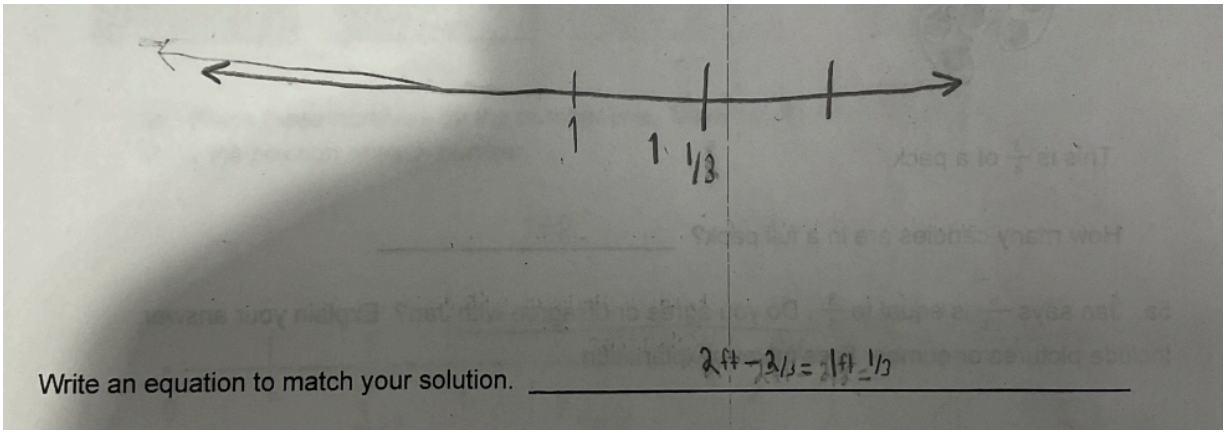
Scoring Guide:

- Give one point for a correct solution. Student may draw pictures, or apply another strategy to solve the problem. Accept either $1\frac{1}{3}$ *or* $\frac{4}{3}$ as a correct answer.
- Give one point for a correct equation: $2 - \frac{2}{3} = 1\frac{1}{3}$ (Credit can be given also for a partially correct equation (e.g. 2 - ⅔ = 1½) In this case the student correctly sees the calculation the problem solves for, but is incorrect in their computation.

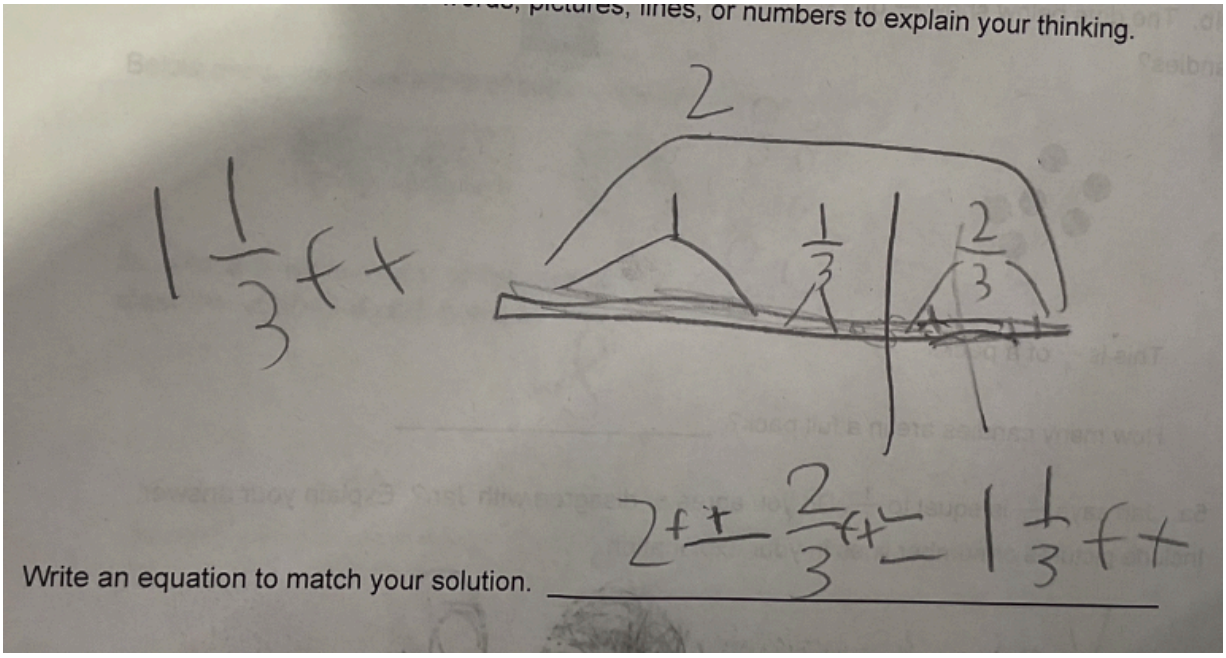
The sample below should receive one point for a correct answer, but no point for an equation.



This answer should receive 2 points. One for a correct answer, one for the correct equation.



Another 2 point example



Instructions for the Administration of
The 5/6 Fractional Reasoning Screening Assessment
April 16, 2024

This assessment is intended to be administered to the whole class at once. The teacher will be asked to read a script in order to establish a pace for the assessment and keep the time demands minimal. There will be students who are unable to complete any of the tasks during the time provided, plan to provide the time at the end of the assessment or at a later time.

When should this assessment be used? This assessment has been designed to be used at one of the three times below.

Mid 5th Grade: Administer in the midst of 5th grade fractions instruction as a formative assessment.

End of 5th Grade: Administer the assessment after the conclusion of instruction related to fractions in 5th grade to identify areas for re-instruction, additional practice, and small group work.

QR Codes for Videos

Early 6th Grade: Use this assessment before or early in the instruction of fractions and proportions in 6th grade to gather formative information to inform topics and students for pre-teaching before formal instruction with fractions begins.

Time: Plan for **60 minutes** for the whole group administration and additional time for extended time for targeted students. Some classes will finish in as little as 45 minutes.

Preparation:

- Photo copies of the assessment for all students
- Pencils with erasers (not pens) for all students
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the video: There are two videos that accompany this assessment for questions 6a and 6b. Have the videos cued before you begin the assessment.

Assessment administration:
Please follow the script as written and have a timer ready. This will help to keep the assessments more reliable. It will also ensure that the assessment remains efficient.

Answer questions only as necessary to ensure students understand the task. Refrain from answering questions that would be leading, influence student thinking, or provide clues to how to solve the tasks.

Active Proctoring: Teachers are encouraged to move about the room and to interact appropriately and actively with the students.

- Where there are students who seem to not be engaging, it is appropriate to encourage them, and to re-read the question for them to ensure understanding.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning.
- When you see student work that leaves you uncertain about their reasoning, ask them questions and encourage them to draw, use number lines, or demonstrate their thinking in other ways.

Forefront users: Forefront has been configured to support data collection for Forefront clients. If you are interested, please reach out to support@forefront.education to request that the assessments be added to your district's resources.

Observations: While systemic collection of observational data is not included in this assessment look-fors are called out in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring: Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the students to ensure the accuracy of the analysis.

Accommodations: Any special accommodations which would normally be given for individual students should also be given for this assessment.

Grade 5/6 Script: Fractional Reasoning Screener

Please read these instructions completely to familiarize yourself with the instructions in preparation for the assessment passing out the papers. Teachers should read all of the bolded text aloud to the students.

Before you begin the assessment, prepare the videos to be shown for tasks 6a and 6b.
6a video 6b video

Teacher: **Today we are going to solve some problems to help me understand how you understand fractions. The results of this assessment will not be part of your grade. This is only to inform my teaching and to help me teach you better.**

This is an opportunity for me to learn how you think about these kinds of questions, so it will be important for you to show your thinking on each of the tasks, with words, pictures, numberlines, and numbers.

Please keep your eyes focused on your own paper. It is important that I get to know your own thinking about these ideas so that I can support you better. (Some teachers may want to spread students out a bit, and/or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please do not work ahead, since these questions are a little different than other assessments. The last question will include two videos we will watch together.

As you are solving the problems there will always be space for you to make drawings and number lines and write words. Please show your thinking as well as you can to explain your answers and justify your answers.

If at any time you have a question, please raise your hand. Are there any questions before we get started? (answer questions as necessary)

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

Task 1:

When students are ready, read the script (Read everything in bold). Pause after each of the numbers to give students the opportunity to write their numbers. Continue at a reasonable pace. You may say the numbers more than once if necessary.

Teacher: **For the first question I will read some numbers aloud. Write the numbers in each of the boxes.**

In box A write the number: one and one third.
In box B write the number: three tenths.
In box C write the number: eight thirds.

Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Task 2a:

Teacher: **Number two a.**

For each of the fractions below write a fraction for the shaded part

Begin.

Allow 2 minutes for this question (continue along earlier if everyone has finished). After 2 minutes, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Continue to task 2b.

Task 2b
Teacher: **Number two b.**

This gray rectangle is two fifths of a whole rectangle. In the space below draw a whole rectangle.

You may make marks to help you. If you want to, you can also draw on the gray rectangle to show your thinking.

Begin.

Note: Do not provide, or allow students to use, rulers. If a student asks to use a ruler or some other length measurement device, say, **"That is not a bad idea, but just estimate for now. It does not need to be perfect."**

Allow 5 minutes for this question (continue along earlier if almost everyone has finished). After 5 minutes, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Task 3a
Teacher: **Number three a.**

Teacher: **Here is a number line. It has the numbers 0 to 5 already on it. Put these numbers onto the number line. (Do not read the fractions.) Make a mark on the line where each number belongs, and write the number by the mark.**

Circulate the room to ensure that students are clearly marking their number lines and, if necessary, remind them to write the fraction to indicate which fraction goes with which mark.

2 Minutes: Allow 2 minutes for this question (continue along earlier if everyone has finished).

If there are students who have not had enough time, say, **"We are going on to the next question. I will give you time to work some more later."**

Task 3b

Teacher: **Number three b. The line below has 0 and 1 already marked on it. There are also some fractions, but they are missing either their numerators or their denominators. Put numbers into the boxes to complete the fractions so that they make sense.**

3 Minutes: Allow 3 minutes for this question (continue along earlier if everyone has finished).

Task 4a

Teacher: **Number four a. The circles below show a package of ten candies. I want to give one fifth of the package of candies to my friends. Show how many of the candies I want to give my friend.**

How many candies are in one fifth of the package?

Show your thinking in the space, and write your answer in the space provided.

Students may either draw to show how many candies, or may indicate in the box of candies by circling their ideas. For students who indicate quickly their response, encourage them to show their thinking in the space provided.

Remind students to write their numeric answer in the space provided if they have not.

3 minutes: Proceed once everyone has answered, or allow up to 3 minutes.

Task 4b

Teacher: **Number four b. Joseph has a collection of cars. He puts 8 of his cars on the table and says, "This is two thirds of my collection. How many cars are in his full collection?**

Circulate to encourage students to explain their reasoning in the space provided and/or by drawing on the image.

3 minutes: Proceed once everyone has answered, or allow up to 3 minutes.

Task 5a
Teacher: **Number five a. Use greater than, equal to, and less than signs to compare each pair of fractions. Explain your answers with drawings, number lines, and/or words.**

Circulate to observe students working and to encourage them to make drawings for each comparison. This might be rectangles, number lines, circles. Students may also want to use numbers to help them make the comparisons. This is good too. The drawings will not be scored (only the comparisons), so encourage students to communicate their thinking for your formative information.

5 minutes: Proceed once everyone has answered, or allow up to 5 minutes.

Task 5b
Teacher: **Number five b. Write three fractions that are equivalent to four sixths. Show your thinking in numbers, words, or pictures.**

3 minutes: Proceed once everyone has answered, or allow up to 3 minutes.

Task 6a
Teacher: **Number six a. For this question I will read it, and then we will watch a short video that goes with the question. I will tell you when to start after we watch the video.**

A strip of paper is two and one half inches long. I cut off three fourths of an inch. How much of the strip is left? Solve in the space below and circle your answer.

Below it says, "Write an equation to match your solution."

Let's watch the video. Play the video: Video Link

After the video say, **Solve the problem in the space provided, circle your answer, and write an equation that goes with this problem.**

5 minutes: Proceed once everyone has answered, or allow up to 5 minutes.

Task 6b
Teacher: **Number six b. For this question I will read it, and then we will watch another short video that goes with the question. I will tell you when to start after we watch the video.**

I have 3 cups of rice. I am using a cup to make servings of one quarter cup each. I want to use all of the rice. How many plates will I fill with one quarter cup each? Solve in the space below and circle your answer.

Below it says, “Write an equation to match your solution.”

Let’s watch the video. *Video Link*

After the video, say, “**Solve the problem in the space provided, circle your answer, and write an equation that goes with this problem.”**

Videos may be shown more than once.

5 minutes: Proceed once everyone has answered, or allow up to 5 minutes.

That is the last question. Take some time now to look over your answers and complete any questions you haven’t finished.

Conclude the assessment. As you collect the papers ensure that all the questions are answered and ask students to finish things as necessary. If there were students who needed more time, provide it now, or later. See instructions related to diagnostic assessment for students with more than a few incomplete answers.

5 / 6 Fractional Reasoning - Show what you know.

Name _____ Date _____

1. Write the numbers in each of the boxes as your teacher reads them.

A	B	C

2a. For each of the shapes below write a fraction for the shaded part.

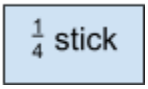
2b. This rectangle represents a piece of chocolate. This piece is $\frac{2}{5}$ of the whole chocolate bar.

In the space below draw a whole chocolate bar.

2c. The rectangle below represents $\frac{1}{4}$ stick of butter.

You need $\frac{3}{2}$ sticks of butter for a recipe.

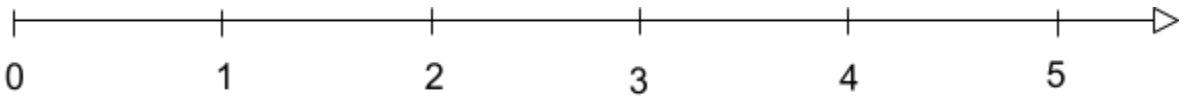
Add to the rectangle to make $\frac{3}{2}$ sticks of butter.



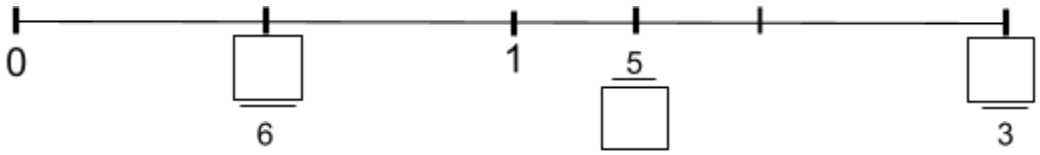
3a. Here is a number line. It has the numbers 0 - 5 already on it.
Put these numbers onto the number line.

$\frac{1}{3}$ $\frac{2}{3}$

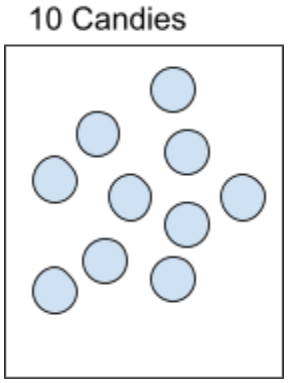
Make a mark where the number belongs, and write the number by each mark.



3b. The line below has 0 and 1 already marked on it. There are also some fractions, but they are missing either their numerators or their denominators. Put numbers into the boxes to complete the fractions so that they make sense.



4a. The circles below show a package of 10 candies. I want to give $\frac{1}{5}$ of the package of candies to my friend. Show how many of the candies I want to give my friend.



How many candies are in $\frac{1}{5}$ of the package? _____

4b. Joseph has a collection of cars. He puts 8 of his cars on the table and says, "This is $\frac{2}{3}$ of my collection." How many cars are in his full collection? Show your thinking.

Joseph's full collection has _____ cars.

5a. Use >, =, < to compare each pair of fractions. Explain your answers with drawings, number lines, and/or words.

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{3}{2}$ $\frac{2}{3}$ $\frac{2}{6}$ $\frac{3}{9}$

5b. Write three fractions that are equivalent to $\frac{4}{6}$.
Show your thinking in numbers, words, or pictures.

6a. A strip of paper is $2\frac{1}{2}$ in. long. I cut off $\frac{3}{4}$ in. How much of the strip is left?
Solve in the space below and circle your answer.

Write an equation to match your solution. _____

6b. I have 3 cups of rice. I am using a one fourth cup to measure servings onto plates. I want to use all of the rice. How many plates will I fill with one quarter cup each? Solve in the space below and circle your answer.

Write an equation to match your solution. _____

Fractional Reasoning Screener
5 / 6 Scoring Guide

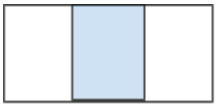
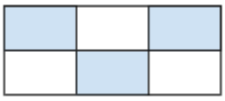
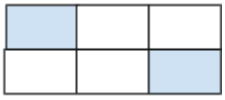
Forefront data entry: Enter results as indicated for each of the questions.

Observations: For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These should be taken into consideration during the scoring.

1. Writing Fractions from Dictation — FR.4.WS (Words and Symbols)

Possible points: 3, one point for each correctly written fraction Enter points into Forefront. Performance Levels: ● All accurate (3 points) ● Not yet (< 3 points)
Answer Key: $A.\ 1\frac{1}{3}$ $B.\ \frac{3}{10}$ $C.\ \frac{8}{3}$
Clarifications: Credit should be given if the number is legible and accurate. While students should be encouraged to write the numerals vertically, give points if they have written them like this 1 ⅓.
Readiness expectation: All correct

2a. Given ⅗ of a rectangle, show the whole rectangle — FR.5.Shapes (Shapes)

Points: Give one point for each correct fraction. Performance Levels: All correct (5 points) - Meets expectations 3 - 4 points - Approaching expectations 0 - 2 points - Not yet
 ⅓  ¼  ½ or 3/6  2/6 or ⅓
Readiness expectation: Students need to recognize that fractions need to have equal sized pieces, and that those pieces do not need to be contiguous. This task represents a low level of understanding and is intended to identify students and groups of students who would benefit from review.

2b. Given ⅖ of a rectangle, show the whole rectangle FR.5.Shapes (Shapes)

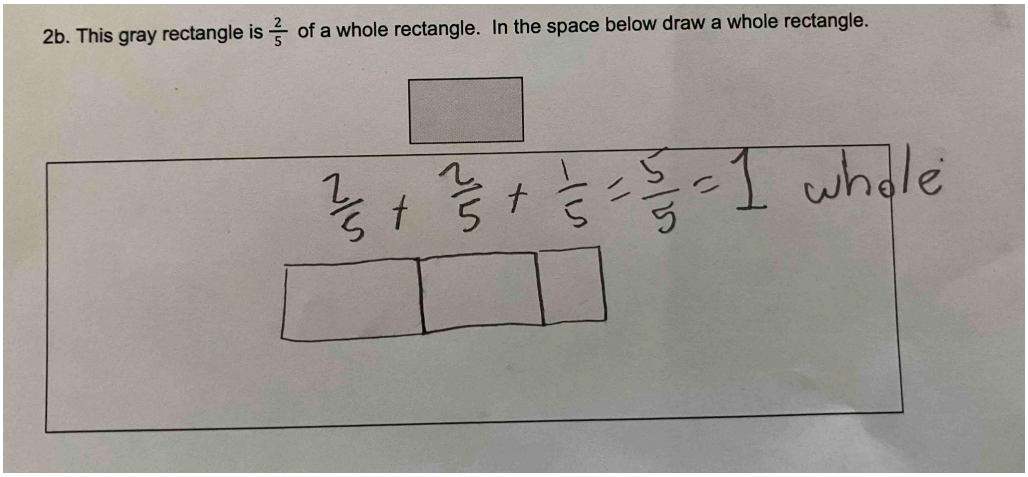
Performance Levels: Enter 1 or 0 into Forefront

- Shows accurate reasoning (1) - student accurately draws a rectangle that is two fifths larger than the gray rectangle and demonstrates partitioning and iterating.
- Not yet (0)

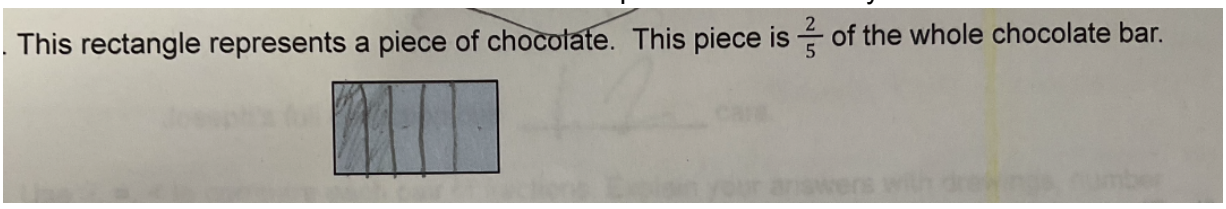
Readiness expectation:
Students may reason through this in a variety of ways, but the most common to look for is when students have reasoned that a total of five fifths need to be constructed. Students should show a recognition of the unit of ⅕ as half the size of the given rectangle and to then create the whole rectangle, the fifths then need to be repeated (iterated) 5 times.

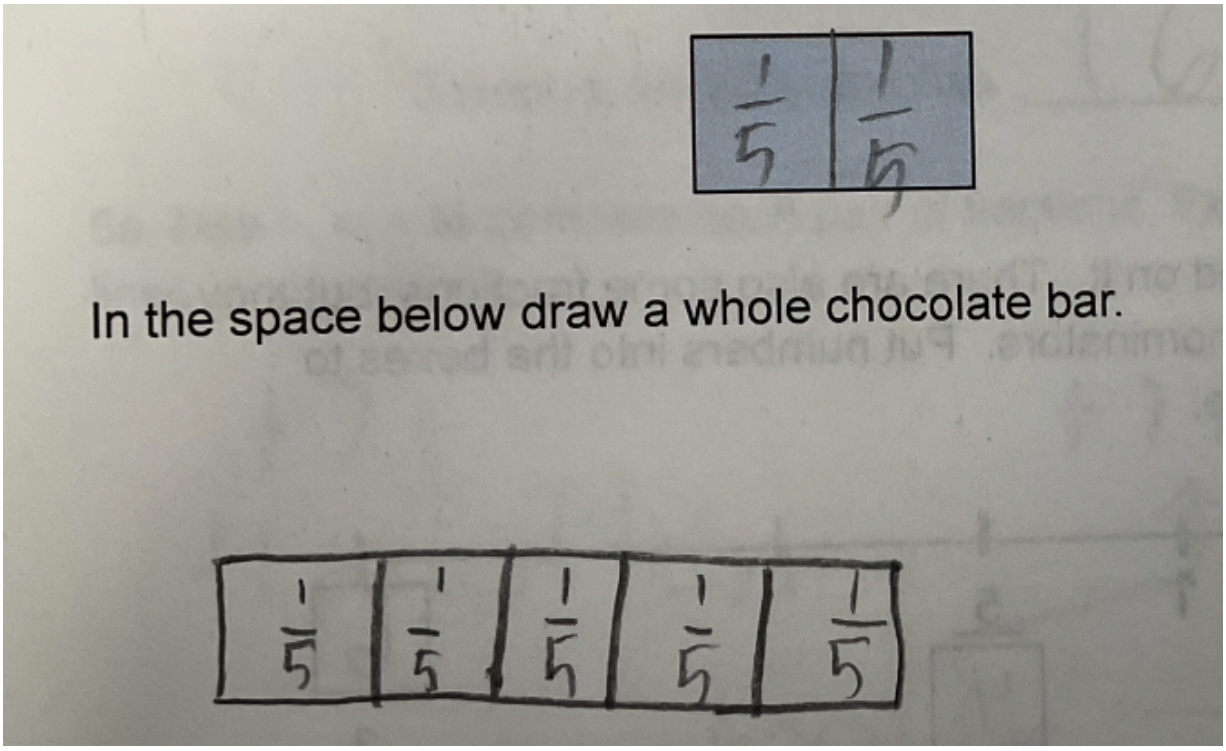
As you circulate the room during the administration of the task, look for students who are attempting to partition the rectangle into 2 equal parts, using their fingers, or pencils, etc. (see note about not using rulers below).

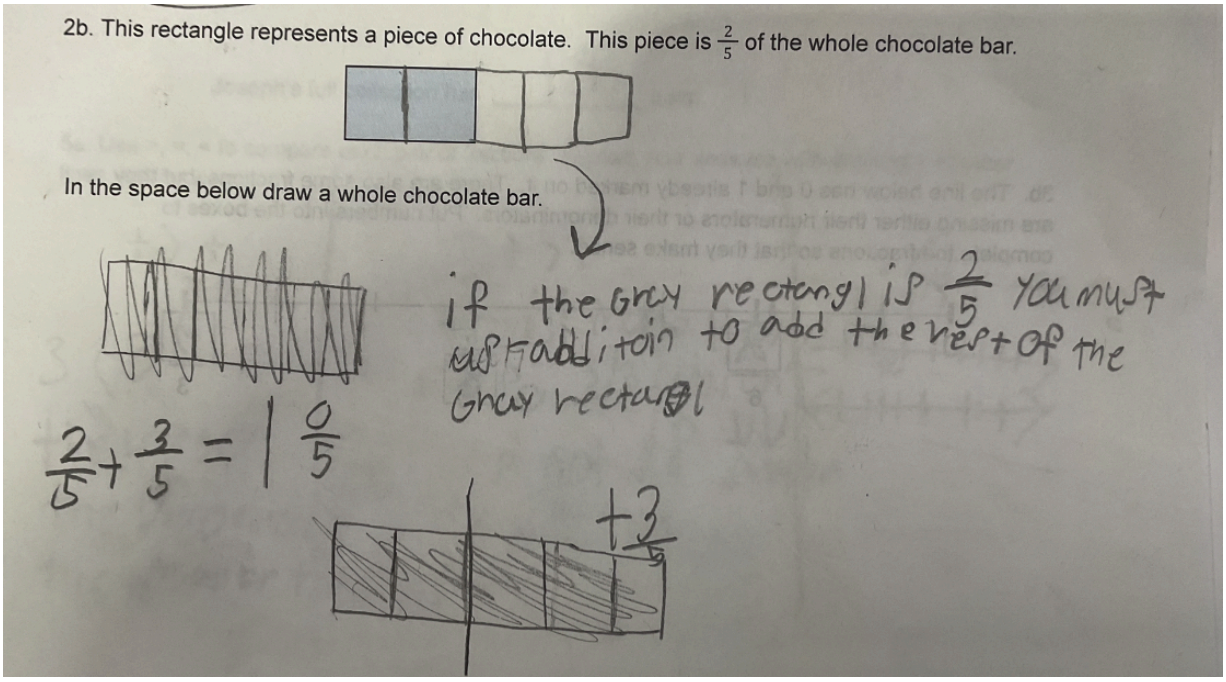
Precision is not of primary importance. Look for evidence of reasoning. When possible, ask students to explain their thinking if it is not obvious.



A Common Misunderstanding to watch out for:
Many students will want to divide the chocolate into 5 equal pieces, not recognizing that in this situation the numerator indicates the number of partitions necessary to find the unit fraction.





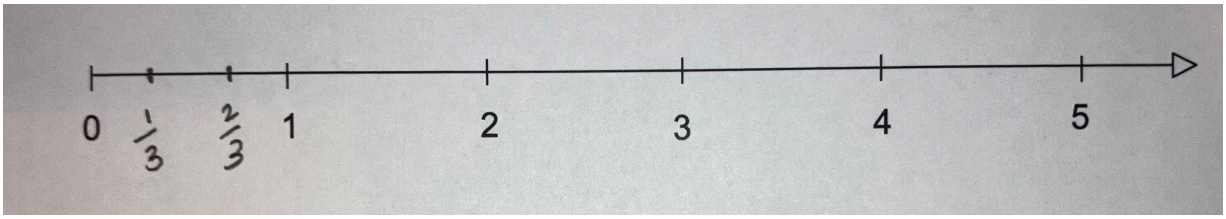


Commentary: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long. We can see that a student applies partitioning and iterating in their response without the need for precise measurements.

2c. Three halves of butter.

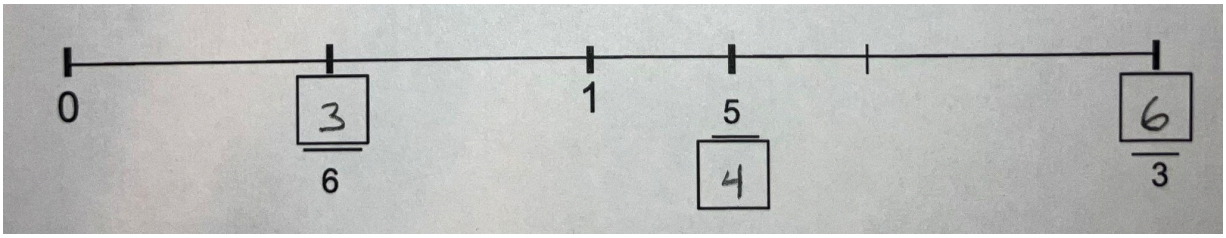
Points Possible: Enter 1 or 0 in Forefront Meets Expectations: 1 point Not Yet: 0 points
Scoring Guide: Give one point for a response that clearly shows that the quarter length from the sample needs to be iterated 6 times to build 3 halves.
Readiness Expectation: Students show that they understand that two quarters are needed to make a half. This is a simple equivalency, that students should be able to to demonstrate using the geometric shape (a rectangle.) Additionally, students should demonstrate that the halves need to be iterated 3 times to make the fraction requested. Students may show that they have created one whole followed by one more half.

3a. Fractions on Number Lines FR.5. Lines (Shapes)

Points possible: 2 points (enter points in Forefront) Performance levels: Meets Expectations: 2 points Not Yet: 0 or 1 point - Not yet
Scoring Guide: Give one point for each correctly placed fraction. Give a point for ⅓ if it is placed left of center between 0 and 1. Give a point for ⅔ if it is placed right of center between 0 and 1. 
Readiness expectation: Students should be able to place fractions on number lines with reasonable accuracy. The obvious distraction of having the line to 5 allows for students to reveal their understanding of this. If a student has placed the fractions in incorrect positions, ask them to explain their thinking for formative purposes.

3b. Missing Numerators and Denominators FR.4.NL (Number Lines)

Points possible: 3 points (Enter points in Forefront) Performance Levels: Meets Expectations: 3 points Not yet: 0-2 points
Scoring Guide: Give one point for each correct response.



Readiness expectation:
Students should be able to apply their understanding of the meaning of numerators and denominators to reason about what must be the missing part of each of these fractions.

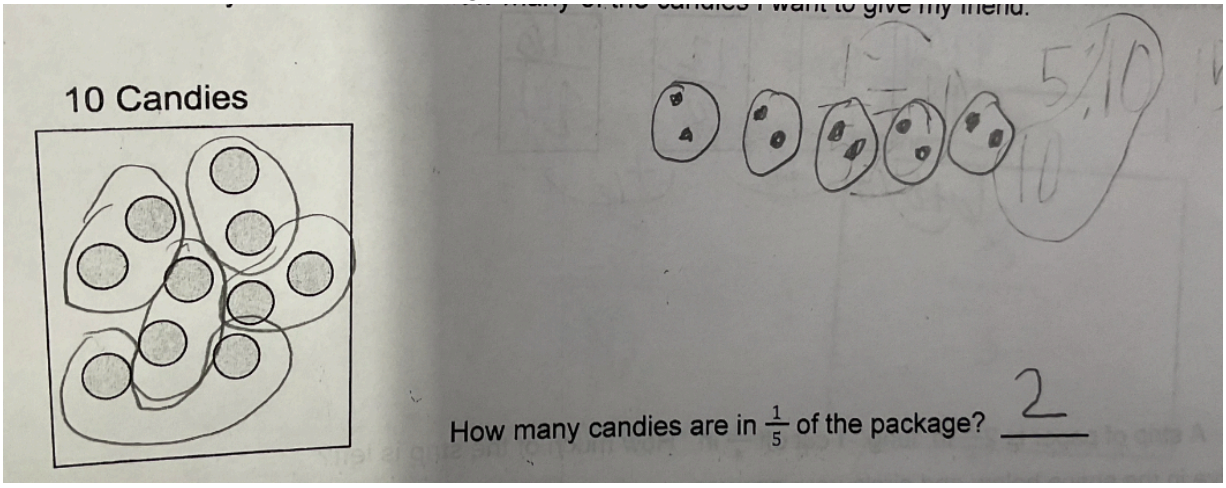
Commentary:

4a. ⅕ of a package

Performance Levels: Enter 1 or 0 in Forefront

- Meets expectations (1) : Student is able to divide the set into 5 equal groups and correctly respond that 2 candies would be ⅕ of the package
- Not Yet: Student does not identify that 2 candies is ⅕ of the set

Scoring Guidance: Look for students to divide the candies into 5 equal groups, either by circling or drawing the set. While the question will be scored using the correctness of the answer, teachers are encouraged to ask students to explain their reasoning to help understand how students are arriving at their answers, whether correct or incorrect.



For students who answer quickly and accurately, consider asking them if they can write an equation to match the situation.

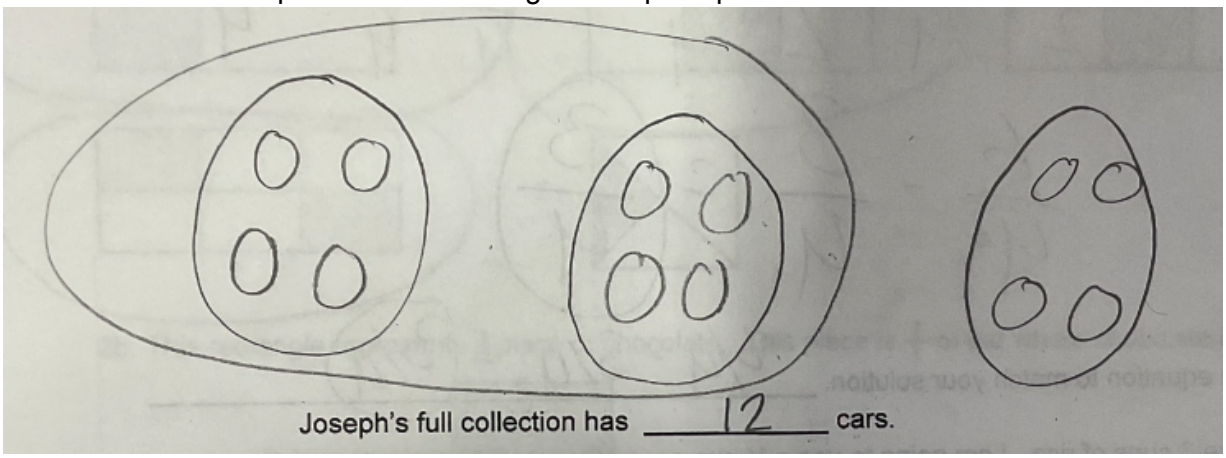
Readiness expectations: This question, if tied to an expression, would be ⅓ * 6 or 6 * ⅓ which aligns with 5th grade expectations for fractions in most states. However, the ability to understand that generating fractions means creating equal partitions aligns with 3rd grade expectations. To be fully ready for the work of 6th grade students should be able to identify a fraction of a set, and also to connect this understanding to multiplication.

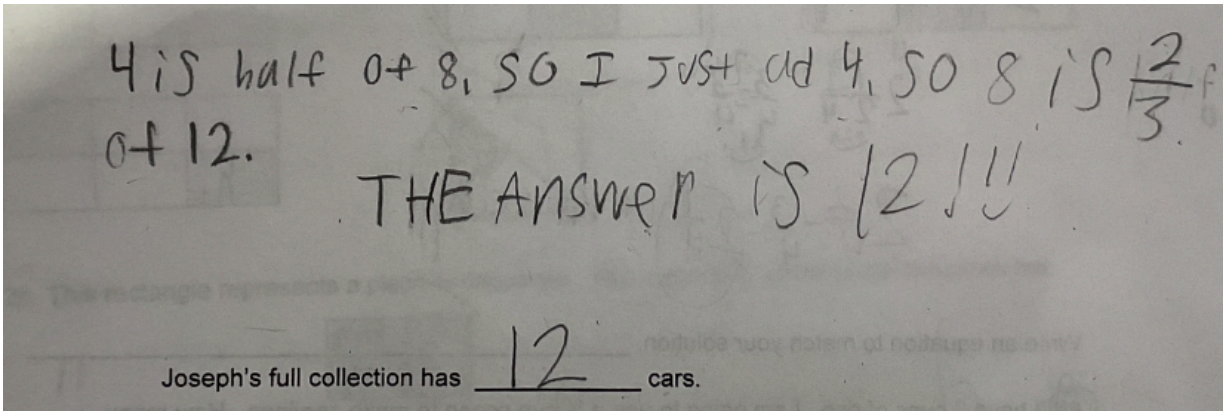
4b. 8 is two thirds. How much is 1? FR.6.Sets (Sets)

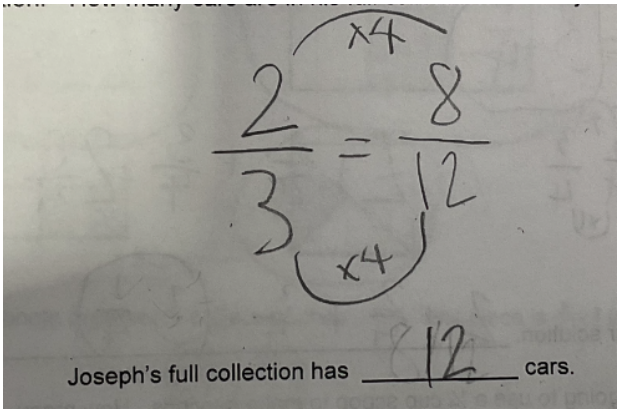
Performance Levels: Enter 1 or 0 in Forefront

- Meets Expectations: Applies understanding of fractions to sets. Correctly answers 12. (1)
- Not yet (0)

Scoring Guidance:
Students should answer 12 cars in the full collection. Encourage students who only put a numeric answer to explain their reasoning in the space provided.







Readiness expectation:
Students who are ready for the work of 6th grade have an understanding of fractions that allows them to understand that the group of with needs to be divided into to parts as defined by the numerator in ⅔, to determine the group size, and that that group needs to be iterated again to create the whole.

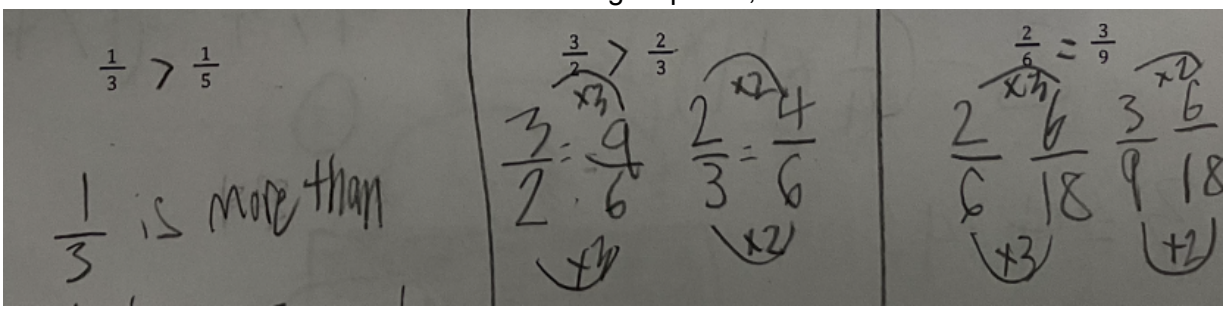
Students are not expected to solve this problem with an equation at this time, though the situation could be modeled with the equation 8 ÷ ⅔. If the student does use this equation, challenge them to make a visual representation to match.

5a. Find equivalent fractions FR.5.MCE (Magnitude, Comparisons & Equivalence)

Performance Levels: Enter points in Forefront.
Meets Expectations (3 points) - student fully demonstrates an ability to reason about the size of fractions in order to compare them.
Not Yet (< 3 points): Student does not fully demonstrate an ability to reason about the size of fractions to compare them

Scoring Guide:

- For each comparison, give 1 point for each correct comparison with clear reasoning about the size and/or the equivalence of the fractions in order to compare them. Score the reasoning. If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<,>,=) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol, take note, and give a point for showing reasoning about size.
- If the student is unable to reason about the size of the fractions, but instead applies some sort of procedure to correctly compare the fractions as the student if they have another way to show how they know their answer is correct. For example, if a student says that they know that ⅓ is greater than ¼ because the denominator is smaller, ask them to make a drawing or use a number line to show how that works.
- If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, numbers, or other means to explain their reasoning about the size of the fractions DO NOT give points, even if the answer is correct.



5b. Find equivalent fractions FR.5.MCE (Magnitude, Comparisons & Equivalence)

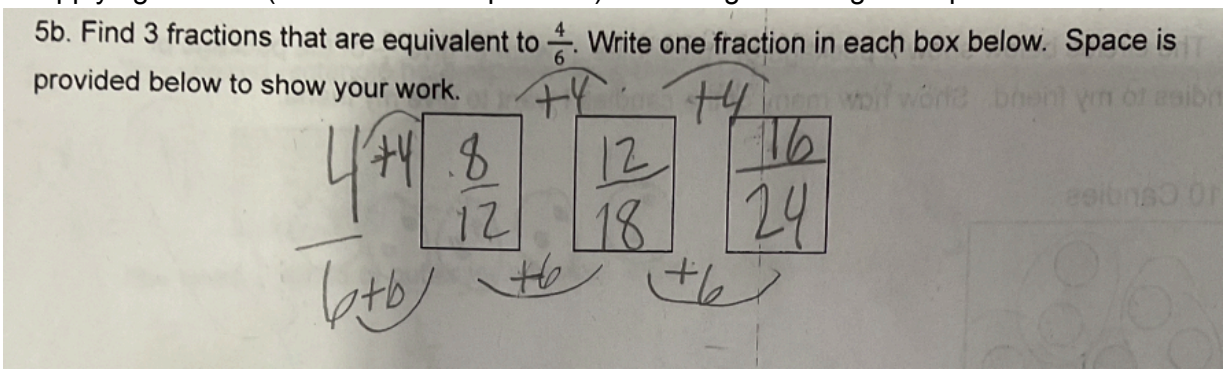
Points possible: 3 (enter points in Forefront)
Performance Levels:

- Demonstrates the ability to generate equivalent fractions (3 points)
- Not yet (<3 points)

Scoring Guide:

- Give one point for each accurate equivalent fraction. Examples are: $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{20}{30}, etc.$

While this answer is correct and should be scored as such, notice that the student appears to be applying additive (rather than multiplicative) reasoning in finding the equivalent fractions.



6a. 2 ½ in. - ¾ in. FR.5.C (Computation)

Points possible: 2 (Enter points in Forefront)
Performance Levels

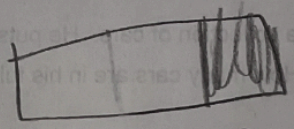
- Meets Expectations (2 pts.): Student shows the ability to solve a simple subtraction problem with fractions and whole numbers in context, and write an equation that matches.
- Approaching Expectations (1 pt.): Student is either able to solve a problem or write an equation to match a problem, but not both.
- Not yet (0 pts.)

Scoring Guide:

- Give one point for a correct solution. Student may draw pictures, or apply another strategy to solve the problem. Accept either $1\frac{3}{4}$ *or* $\frac{7}{4}$ as a correct answer.
- Give one point for a correct equation: $2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$, or some variation thereof.

6a. A strip of paper is $2\frac{1}{2}$ in. long. I cut off $\frac{3}{4}$ in. How much of the strip is left?

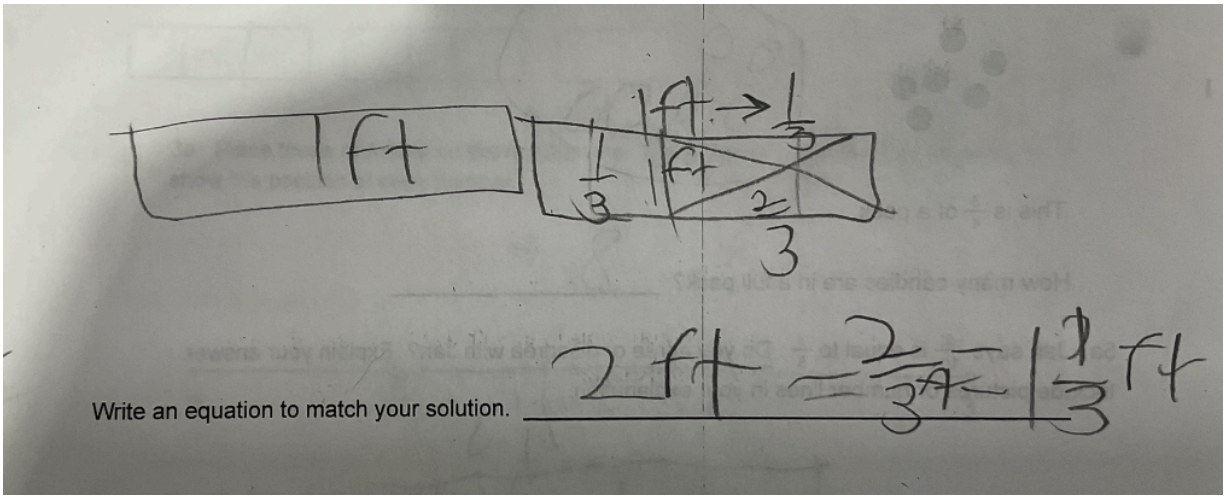
Solve in the space below and circle your answer.



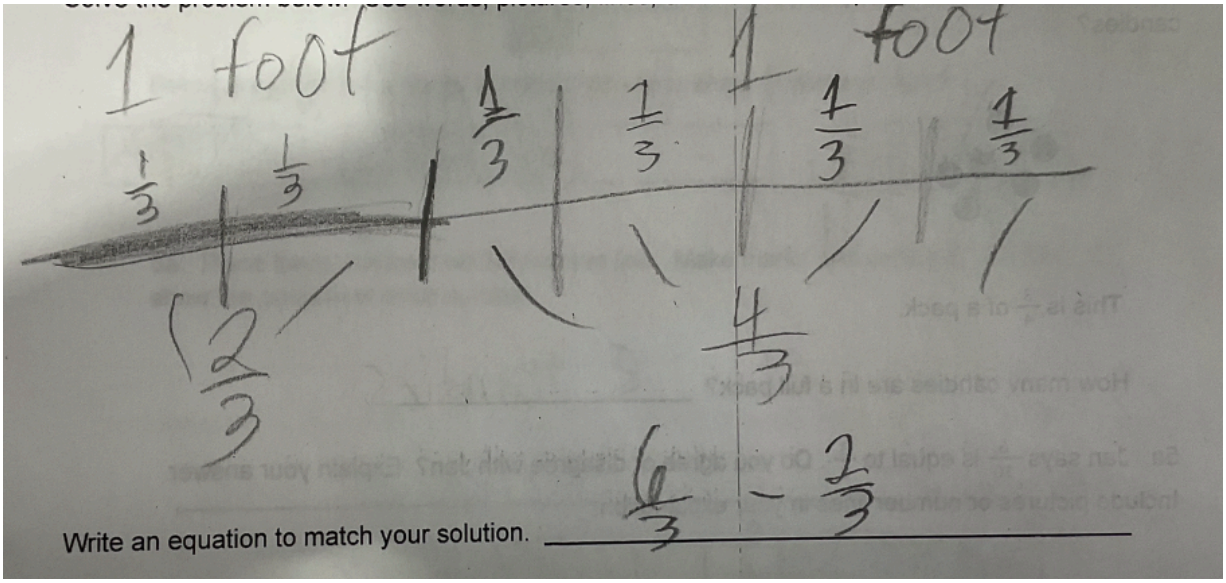
$2\frac{1}{2} - \frac{3}{4} = 1\frac{6}{8}$

$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 2\frac{1}{2}$

Write an equation to match your solution. $2\frac{1}{2} - \frac{3}{4} = 1\frac{6}{8}$



Write an equation to match your solution. $2\text{ ft} - \frac{2}{3} = 1\frac{1}{3}\text{ ft}$



Write an equation to match your solution. $\frac{6}{3} - \frac{2}{3}$

6b. 3 cups divided into quarter cups. FR.5.C (Computation)

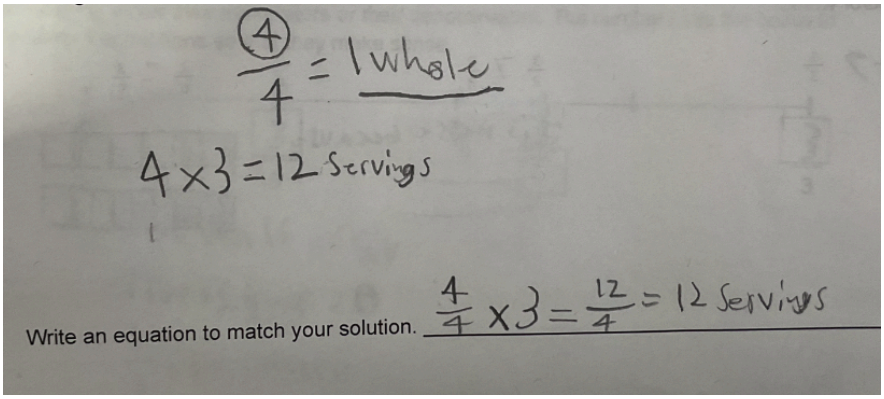
Points possible: 2 (Enter points in Forefront)
Performance Levels

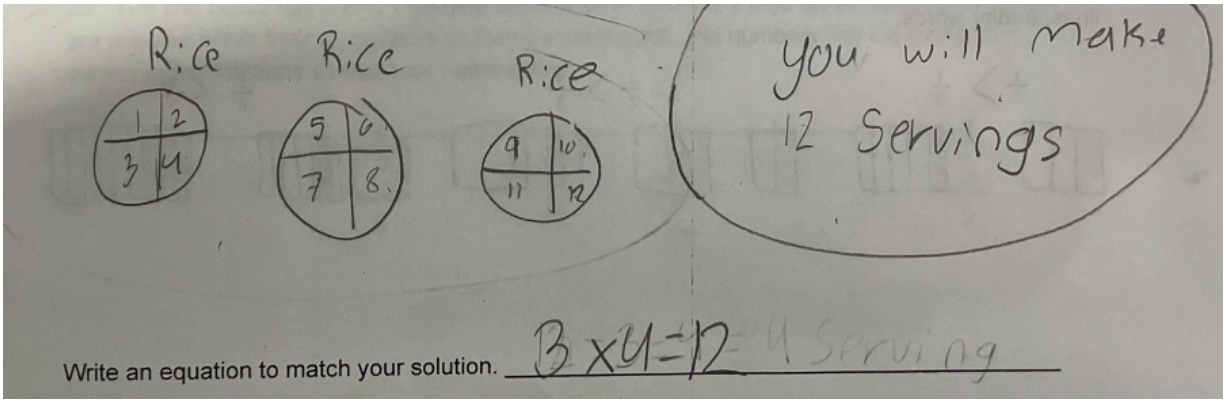
- Meets Expectations (2 pts.): Student shows the ability to solve the problem and create an equation to match.
- Approaching Expectations (1 pt.): Student is either able to solve a problem or write an equation to match a problem, but not both.
- Not yet: solution and equation both incorrect (0 pts.)

Scoring Guide:

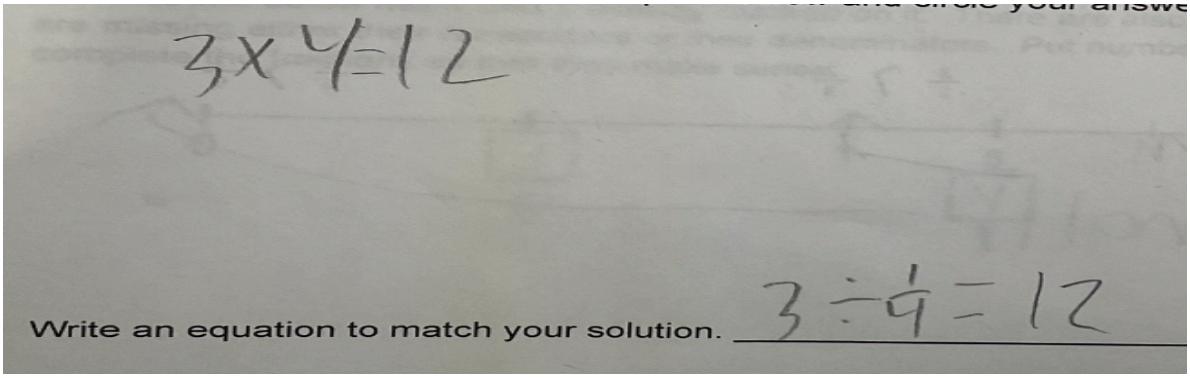
- Give one point for a correct solution of 12.
- Give one point for a correct equation: $3 \div \frac{1}{4} = 12$, or some variation thereof.

Examples of responses that should be scored as Meets Expectations (2)

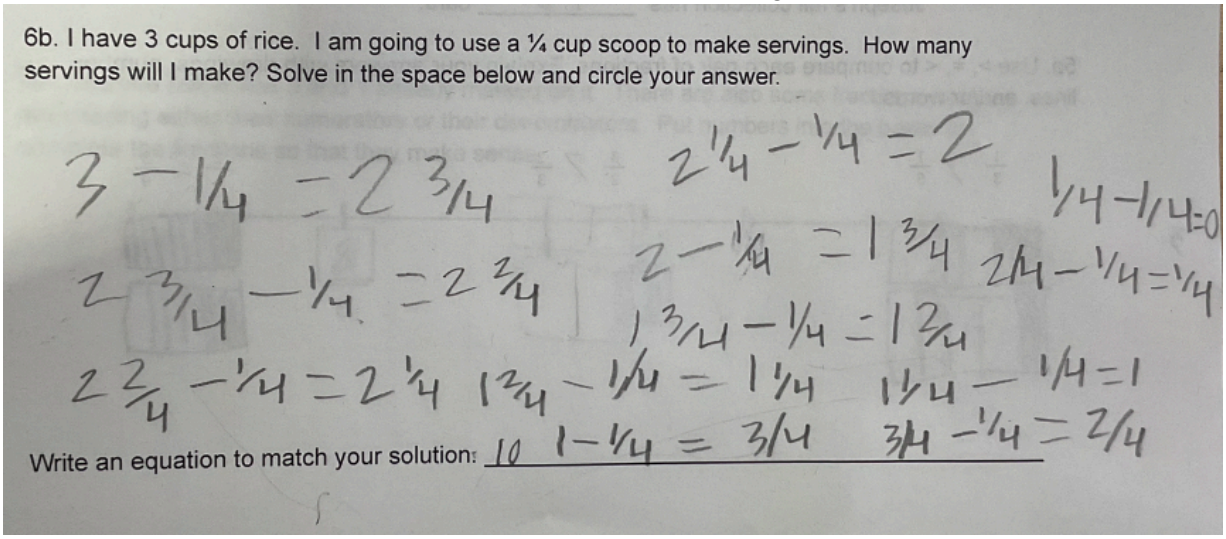




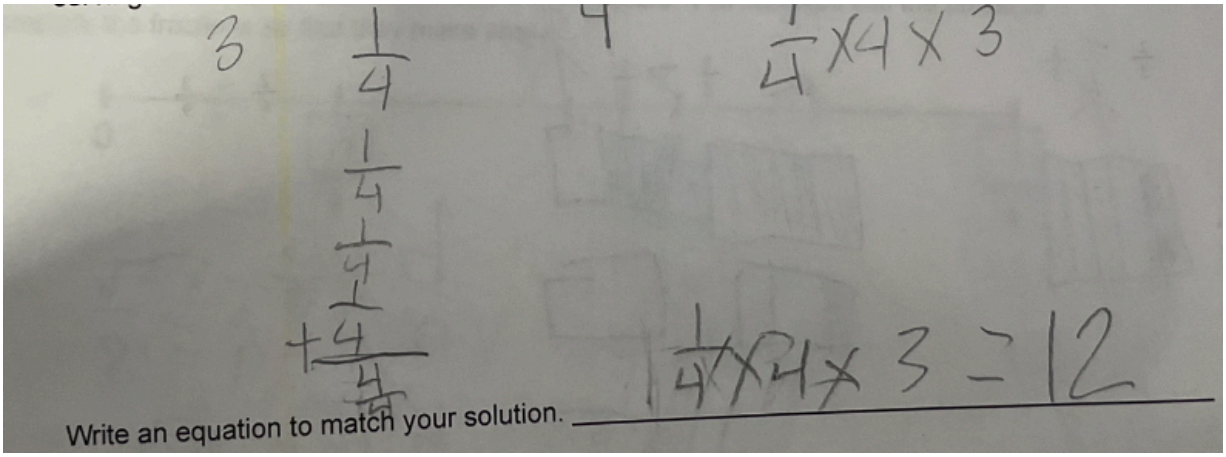
While it would have been good to ask this student to elaborate on their thinking with a drawing or something to demonstrate their thinking, this response should be scored as Shows Reasoning (2).



This response below, though not fully correct (they appear to have answered 10) they have shown that this problem can be solved with repeated subtraction and have created a series of equations to do so, and so should be scored as Approaching Expectations (1)



The example below should be scored as Approaching since the student has a correct answer of 12, but an inaccurate equation.



Fractional Reasoning Assessments
Diagnostic Interview

Introduction
The Fractional Reasoning Diagnostic Assessment is an interview-based assessment that teachers adapt for each student. Teachers will want to start with one of the Fractional Reasoning Screener assessments to get initial information. The screeners can be a starting point for the diagnostic assessment especially if a teacher talks directly with a student as they attempt the problems on the screening assessment (or after the assessment is administered) to gather more information about how the student thinks about those problems across the different facets. This diagnostic assessment is designed to help teachers identify a performance level (Level Pre, to Level D) for each of the 6 facets of fractional reasoning (as necessary).

What is meant by "diagnostic" assessment?
This assessment is diagnostic in that it is intended to provide detailed, precise, and complete information about the student's understanding of and ability to reason with fractions to inform instruction and help teachers understand when to strategically use scaffolds and supports to help students access grade level content. This assessment is not intended to place students in percentile bands or identify eligibility for special education services.

Ceiling/Floor Assessment
The Fractional Reasoning Diagnostic assessment is a "ceiling/floor" assessment by design. That is, the assessment administrator will utilize a series of tasks each of which have a variety of subtasks that vary in difficulty. Teachers should use their prior understanding of the student and results from the screening assessments to select which level of difficulty to start the assessment. If students are not yet successful with the task, move to an easier one. If students are successful, continue to the more difficult tasks until you have identified a performance level for that facet or reasoning.

Be Flexible for Your Needs
It could be that the teacher will want to choose only one or two specific facets, or facets in their assessment. For example, if a student performs disproportionately low in one of the facets, the teacher might want to administer tasks from that one facet to better understand how the student is conceptualizing fractions.

Fractional Reasoning Diagnostic Assessment
FACET 1: Words and Symbols

Instructions:

1. Prepare the cards and copies of the note catcher.
2. Choose a starting point.
   Recommended starting points -
   - Fall of 4th grade - Part 2
   - Fall of 5th grade - Part 3
3. Administer the tasks as described.
4. Use the chart to determine a performance level
5. Once enough evidence has been collected to determine a level continue to FACET 2.

Determine a Performance Level

Level Pre	Level A Equal partitions	Level B - Unit Fractions	Level C - Proper Fractions	Level D - Operational
Part 1:Not yet	Part 1: All correct Part 2: One correct or Not yet	Part 1: All correct Part 2: Two or three correct Part 3: Not yet	Part 2: Two or three correct Part 3: All correct	Not applicable

Assessment note catcher
Print 1 per student

FACET 1: Fractional Reasoning Assessment
Words and Symbols

Name of student_____ Date _____

Name of test administrator _____ PERFORMANCE LEVEL: _____

Part 1 - Lay card WS1.1 in front of the student. "Here is a circle. How much of this circle is shaded?" Point to the shaded part.
Repeat for cards Facet 1: Card 1 to Facet 1: Card 3

▢ All Correct ▢ Not yet	Notes:   
Scoring Guidance	**Correct answers are, "One fourth" (or one quarter), "One half", and "one third."** If the student says something like "one out of four," ask the student "Do you have another way to say what this part is?"

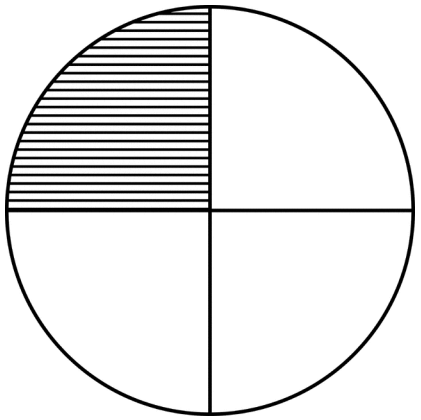
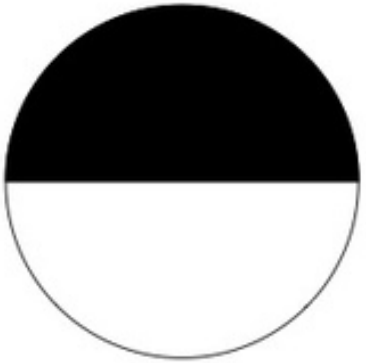
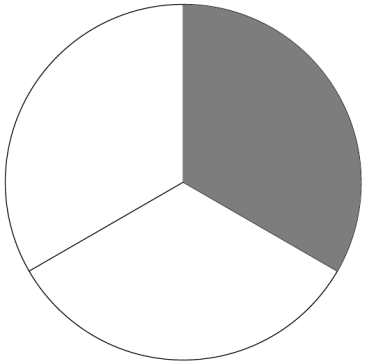
Part 2 - Lay the card WS2.1 in front of the student. "Read this number." Repeat for Cards Facet 1: Card 4 to Facet1: Card 7.

▢ 3 correct ▢ 2 correct ▢ 1 correct ▢ Not yet	Notes: $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{10}$
Scoring Guidance	**Correct answers are, "One third," "One half," and "One tenth."** If the student says something like "one out of three," or, "one over 3," ask the student, "Do you have another way to say this number?"

Part 3 - Lay the cards Facet 1: Cards 8 - 12 in front of the student one at a time. Say, “Read this number.” Repeat for the remainder of the set.

▢ All Correct ▢ Not yet	WS3.1 $\frac{4}{5}$	WS3.2 $4\frac{2}{3}$	WS3.3 $\frac{6}{8}$	WS3.4 $1\frac{1}{2}$	WS3.5 $\frac{7}{4}$
Scoring Guidance	**four fifths**	**four and two thirds**	**six eights**	**one and one half or one and a half**	**seven fourths or seven quarters**
	If the student says something like “four out of five,” or, “one four over five,” ask the student, “Do you have another way to say this number?”				

Cards for Words and Symbols: Part 1

Facet 1: Card 1	Facet 1: Card 2	Facet 1: Card 3
		

Cards for Words and Symbols: Part 2

Facet 1: Card 4	Facet 1: Card 5	Facet 1: Card 6
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{10}$

Cards for Words and Symbols: Part 3

Facet 1: Card 7	Facet 1: Card 8	Facet 1: Card 9	Facet 1: Card 10	Facet 1: Card 11
$\frac{4}{5}$	$4\frac{2}{3}$	$\frac{6}{8}$	$1\frac{1}{2}$	$\frac{7}{4}$

Fractional Reasoning Diagnostic Assessment
FACET 2: Units - Shapes

Instructions:

1. Prepare the cards, student answer catcher, and note catcher.
2. Choose a starting point.
   Recommended starting points -
   - Beginning of 4th grade - Part 2
   - Beginning of 5th grade - Part 3
3. Administer the tasks as described.
4. Use the chart to determine a performance level
5. Once enough information has been collected to determine a performance level, continue to FACET 3.

Determine a Performance Level

Level Pre	Level A - Equal partitions	Level B - Unit Fractions	Level C - Fractions	Level D - Operational
Part 1: Not Yet	Part 1: Correct Part 2: Not yet	Part 1: Correct Part 2: Correct Parts 3a or 3b: Not yet	Parts 3a and 3b: Correct Part 4: Not yet	Parts 3a and 3b: Correct Part 4: Correct

Fractional Reasoning Assessment
FACET 2: Units - Shapes

Student_____ Date _____

Test administrator _____ PERFORMANCE LEVEL: _____

Before you start: Give the student the Answer Catcher and have them write their name.

Part 1 - Point to the rectangle in the Part 1 section of the Student Answer Catcher. Say, "Here is a rectangle. This rectangle represents a snack bar. Three friends want to share the snack bar. Show how you could make fair shares for three kids."
Once the student is finished, say, "Show me how you know that the shares are fair."

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response is approximately accurate in the divisions of the rectangle, and the student, either through words or gestures, demonstrates that they intended to use the entire rectangle and make each of the sections identical in size.

Part 2 - Point to the rectangle in section Part 2 on the student answer sheet. Say, "Here is a whole rectangle. Draw one fifth of a whole rectangle." (point out that the number ⅕ is displayed by the rectangle.) Once the student has finished the drawing, say, "How do you know that yours is one 5th of the whole rectangle?"

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response demonstrates that the student has attempted to create a portion that if repeated 5 times will fill the whole rectangle. That is, that it is truly one fifth of the larger rectangle. The student may partition the original rectangle. In their justification of their answer the student shows, through words and/or gestures that one fifth were repeated 5 times it would be the same size as the original whole.

Part 3a - Point to the rectangle in Part 3a of the Student Answer Catcher. Say, “This is one third of a whole rectangle. Draw a whole rectangle.”
When the student has completed their drawing, ask, “How do you know that would be the whole?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response demonstrates that they have attempted to draw a rectangle that is three times the size of the original.

Part 3b - Point to the rectangle in Part 3b of the Student Answer Catcher. Say, “This is a whole rectangle. Draw three fifths of a whole rectangle.” When the student has completed their drawing ask, “How do you know yours is three fifths of the whole?

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response demonstrates an understanding that the original whole needs to be divided into five equal pieces and that three fifths is found through repeating those fifths three times.

Part 4 - Point to rectangle in Part 4 of the Student Answer Catcher. Say, “This is five sixths of a whole rectangle. Draw a whole rectangle.” When the student has completed their drawing ask, “How do you know that is the whole rectangle?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response demonstrates an understanding that the given rectangle needs to be divided into 5 equal pieces, and that one more piece of that same size would need to be added to make the whole.

Name _____ **Date** _____

Student Answer Catcher
Fractional Reasoning Facet 2: Units - Shapes

Part 1

Part 2

$\frac{1}{5}$

Part 3a

$\frac{1}{3}$

Part 3b

$\frac{3}{5}$

Part 4

$\frac{5}{6}$

Fractional Reasoning Diagnostic Assessment
FACET 3: Number Lines

Instructions:

1. Prepare the cards, student answer catcher, and note catcher.
2. Choose a starting point.
   Recommended starting points - or use screener assessment results as a guide
   - Fall of 4th grade - Part 2
   - Fall of 5th grade - Part 3
3. Administer the tasks as described.
4. Use the chart to determine a performance level.

Determine a Performance Level

Level Pre	Level A Equal partitions	Level B Unit Fractions	Level C Fractions	Level D Operational
Part 1: Not Yet	Part 1: Correct Part 2: Not yet	Part 2: Correct Part 3: Not yet	Part 3: Correct Part 4: Not yet	Part 3: Correct Part 4: Correct

Fractional Reasoning Assessment
FACET 3: Units - Lines

Student_____ Date _____

Test administrator _____ PERFORMANCE LEVEL: _____

Before you start: Give the student the Student Answer Catcher and have them write their name.

Part 1 - Point to the number line on the Student Answer Catcher. Say, "Here is a number line. There is a missing number here. What number must that be?"

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer (5) demonstrates that the student understands that the distance between units on a number line is consistent. Therefore, the distances between the first two dots, if repeated three more times would mean that 5 lands in the space. (If the student attempts to make equal divisions and comes up with another number which is close (e.g. 4 ½) count as correct. The student should, through words or gestures, communicate an understanding that the space between each counting number should be consistent in length.

Part 2 - Point to the number line in Part 2 of Student Answer Catcher. Say, "Here is a line that goes from zero to one. Where would this number go? (Place the card that says ⅓ on the table.) Make a mark on the line and write the fraction." After the student has placed ⅓, place the card that says ¼ on the table and ask, "Where would this number go? Make a mark and write the fraction."

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates that the line should be divided into 3 equal sections to place the number ⅓ and into 4 sections to place the number ¼. ⅓ is placed to the right of ¼ and both numbers are left of half way between zero and 1.

Part 3 - Point to the line in Part 3 of the Student Answer Catcher. Say, “Here is a number line that goes from zero to two. Where does this number belong on the line?” (Place the card that says ⅔ on the table.) “Draw a mark on the line and write the number by it.” After they have written where they think ⅔ goes, place the card that says 5/3 on the table. “Where would this number go? Make a mark on the line and write the number to show where it goes.”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response indicates that the distance between 0 and 1 needs to be divided into 3 equal sections to find the thirds and that the number ⅔ should be two of those sections from 0 (or one section to the left of 1), and the number 5/3 should be placed another ⅔ to the right of the number 1 (or one third to the right of 2).

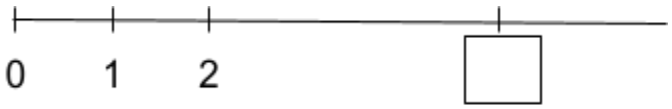
Part 4 - Point to the line in Part 4 of the Student Answer Catcher. Here is a number line. There are two numbers missing from the fractions on the number line. What numbers go in the blanks?

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response has some number between 8 and 13 for the fraction to the left, and the number 3 for the numerator for the fraction between 1 and 2.

Name _____ **Date** _____

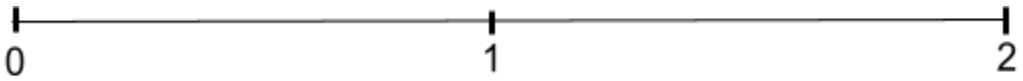
Student Answer Catcher
Fractional Reasoning Facet 3: Units - Lines

Part 1

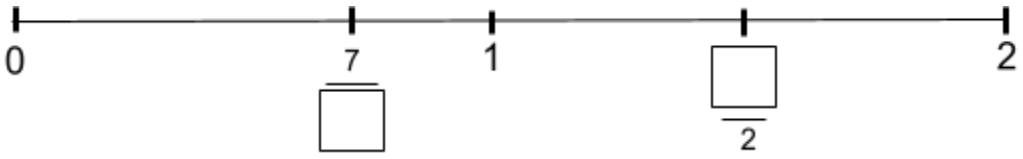


Part 2

Part 3



Part 4



Fractional Reasoning Assessment
FACET 3: Units - Lines

CARDS - Prepare 1 set

$\frac{1}{3}$	$\frac{1}{4}$
$\frac{2}{3}$	$\frac{5}{3}$

Fractional Reasoning Diagnostic Assessment
FACET 4: Sets

Instructions:

1. Prepare the cards, note catcher, and a set of at least 15 counters of the same color.
2. Choose a starting point.
   Recommended starting points - or use screener results to guide you.
   - Fall of 4th grade - Part 2
   - Fall of 5th grade - Part 3
3. Administer the tasks as described.
4. Use the chart to determine a performance level

Determine a Performance Level

	Level Pre	**Level A - Equal partitions**	**Level B - Unit Fractions**	**Level C - Fractions**	**Level D - Operational**
Rubric	Part 1: Not Yet	Part 1: Correct Part 2: Not yet	Part 2: Correct Part 3: Not yet	Part 3: Correct Part 4: Not yet	Part 3: Correct Part 4: Correct

Fractional Reasoning Assessment
FACET 4: Sets

Student_____ Date _____

Test administrator _____ PERFORMANCE LEVEL: _____

Before you start: Give the student the Student Answer Catcher and have them write their name.

Part 1 - Provide the student with 12 counters. Say, “Let’s pretend these are candies. Three friends want to share these candies. They want to make sure that it is fair, and that each friend gets the same amount. How many candies should each friend get?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates that the student can create three equal groups of 4 counters each. Note: For formative assessment purposes (if a score of “not yet” has been determined) on a paper you may three draw stick figures to represent the friends to determine if this scaffold is enough to lead to success.

Part 2 - Provide the student with a set of 12 counters. Say, “Put one third of the counters in this box. Provide the student with the card for part 2.

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates that the student knows to put 4 counters in the box. Observe the student’s strategies carefully for formative information.

Part 3a - Provide the student with 6 counters. Say, “This is <u>not</u> a whole set. This is two thirds of a set.” Show the card with the fraction ⅔. “How many counters are in a *whole* set? Use these counters to make a whole set.”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response indicates that the student is able to create a set of 9 to represent the whole set.

Part 3b - Prepare 15 counters and put them in front of the student. Say, “Here are 15 counters. What is three fifths of 15?” (Put the card that says ⅗ on the table for the student.)

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response has some number between 8 and 12 for the fraction to the left, and the number 3 for the numerator for the fraction between 1 and 2.

Part 4 - Provide the student with 12 counters (have more counters available for the student to work with.) Say, “Let’s pretend that these are candies. These twelve counters are two thirds of a package of candies.” (Put the card that says ⅔ on the table.) “How many candies are in one and one half packages of candies?”

Paper and pencil for note taking may be provided for the student to work with.
Note: For efficiency's sake, given the complexity of this task, the teacher may want to give the student independent work time and then return to the student once they have had the opportunity to come to a solution to check the answer and probe their reasoning.

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct response indicates that 27 counters would be in 1½ packages. The student should be able to explain that if 12 counters represent ⅔ of a package, then each third of a package is 6 counters, meaning that a whole package would have 18 counters. One half of 18 is 9, and so therefore 18 + 9 = 27.

Fractional Reasoning Assessment
FACET 4: Sets

Cards - prepare one set

Card for Part 2

$\frac{1}{3}$

$\frac{2}{3}$

$\frac{3}{5}$

Fractional Reasoning Diagnostic Assessment
FACET 5: Magnitude, Comparisons & Equivalence

Instructions:

1. Prepare the cards and note catcher.
2. Choose a starting point.
   Recommended starting points -
   - Fall of 4th grade - Part 2
   - Fall of 5th grade - Part 3
3. Administer the tasks as described.
4. Use the chart to determine a performance level

Determine a Performance Level

	Level Pre	Level A - Equal partitions	Level B - Unit Fractions	Level C - Fractions	Level D - Operational
Rubric	Not assessed	Part 1: Not yet	Part 1: Correct Part 2a and 2b: Not yet or one correct	Part 1: Correct Part 2a and 2b: Correct Part 3 and 4: One or none correct	Part 2a and 2b: Correct Part 3: Correct Part 4: Correct

Fractional Reasoning Assessment
FACET 5: Magnitude, Comparisons & Equivalence

Student_____ Date _____

Test administrator _____ PERFORMANCE LEVEL: _____

Part 1 - Show the student card 1 (⅕ and ⅓). Say, “Which fraction is greater?” After the student answers, ask, “How do you know?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that ⅓ is greater than ⅕ because the denominator indicates the number of partitions of a whole and more partitions mean smaller pieces.

Part 2a - Show the student card 2a (⅗ and ¾) Say, “Which fraction is greater?” After the student answers, ask, “How do you know?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that ¾ is greater than ⅗ because the denominator indicates the number of partitions of a whole and more partitions mean smaller pieces.

Part 2b - Show the student card 2b (3/3 and 4/4) Say, “Which fraction is greater?” After the student answers, ask, “How do you know?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that 3/3 and 4/4 are equivalent, because they both represent the partitioning of a whole into either 3 or 4 equal pieces, but the whole is still a whole.

Part 3 - Show the student card 3 (3/7 and ⅔) Say, “Which fraction is greater?” After the student answers, ask, “How do you know?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates that the student knows that ⅔ is greater than 3/7. Explanations can include the use of ½ as a benchmark fraction - 3/7 is less than ½ and ⅔ is greater than ½ therefore 3/7 is less than ⅔. If the student uses a common denominator to find the solution their explanation should include reference to why finding a common denominator makes the comparison of fractions possible. That is, by finding a common denominator this makes the fractional pieces the same size, so that it is possible to compare the numerators to determine the greater fraction. If the student’s explanation refers only to the procedure itself, without demonstrating an understanding of why the procedure works, score as “not yet.”

Part 4 - Janet has ¾ of a gallon of glue. The instructions say she needs 6/8 of a gallon of glue for her art project. Janet says, “I don’t have enough glue.” (provide the cards 4a and 4b)

Do you agree with Janet?

When the student has agreed or disagreed, ask, “How could you show Janet that she [does, or does not] have enough?”

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that Janet does have enough glue because ¾ is equal to 6/8.

Fractional Reasoning Assessment
FACET 5: Magnitude, Comparison & Equivalence

Cards - prepare one set

1 $\frac{1}{5}$	$\frac{1}{3}$
2a $\frac{3}{5}$	$\frac{3}{4}$
2b $\frac{3}{3}$	$\frac{4}{4}$
3 $\frac{3}{7}$	$\frac{2}{3}$
4a Janet has $\frac{3}{4}$ gallon	4b Janet needs $\frac{6}{8}$ gallon

Fractional Reasoning Diagnostic Assessment
FACET 6: Computation

Instructions:

1. Prepare the materials, cards, note catcher and a pair of scissors.
2. Do not provide paper and pencil for the student. All tasks should be solved mentally.
3. Choose a starting point.
   Recommended starting points -
   - Fall of 4th grade - Part 2
   - Fall of 5th grade - Part 3
4. Administer the tasks as described.
5. Use the chart to determine a performance level. Once enough evidence has been collected to determine a level, conclude the assessment.

Determine a Performance Level

	Level Pre	**Level A - Equal partitions**	**Level B - Unit Fractions**	**Level C - Fractions**	**Level D - Operational**
Rubric	Not assessed	Not assessed	Part 2: Correct Part 3: Not yet	Part 3: Correct Part 4: Not yet	Part 3: Correct Part 4: Correct

Fractional Reasoning Assessment
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Student_____ Date _____

Test administrator _____ PERFORMANCE LEVEL: _____

Part 1 - Prepare a strip from the Part 1 materials. Show the whole rectangle to the student. "I have a whole strip. I am going to make 5 equal pieces." With a pencil or pen, use the guide marks at the top of the rectangular strip to draw 4 lines. Show the strip to the student with the fifths demarcated. "I have made fifths. Now I am going to cut one off."

Put the strip out of sight from the student (either under the table, or using a screen) and cut off one fifth. Show the fifth to the student (but not the remainder of the strip). "I have cut off one piece. What would you call this piece?" The student should respond "one fifth" - if the student doesn't respond correctly, take note and tell the student, "This is one fifth of the whole strip."
Ask, "How much of the rectangle do I still have hidden? (motion to wherever the remainder of the rectangle is hidden.)

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding ⅘ of the rectangle remains after the one fifth has been separated from the whole. If the student answers "4" or "4 pieces," Say, "This is one fifth of the strip (pointing again to the one fifth). How much is still hidden?"

Part 2 - Show the student the card C2. Say, "Here is a circle. This part is two fifths of the whole circle. How big is this part?" (Point to the corresponding region of the circle.)

▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that three fifths of the circle is the unlabeled part.

Part 3 - Place card C3 in front of the student. Say, "Here are two students. They each have a bag of rice. This student's bag has two thirds of a pound of rice. This other student's bag also has two thirds of a bag of rice. They need three pounds of rice. If they combine their bags in this large pot will they have three pounds of rice?"

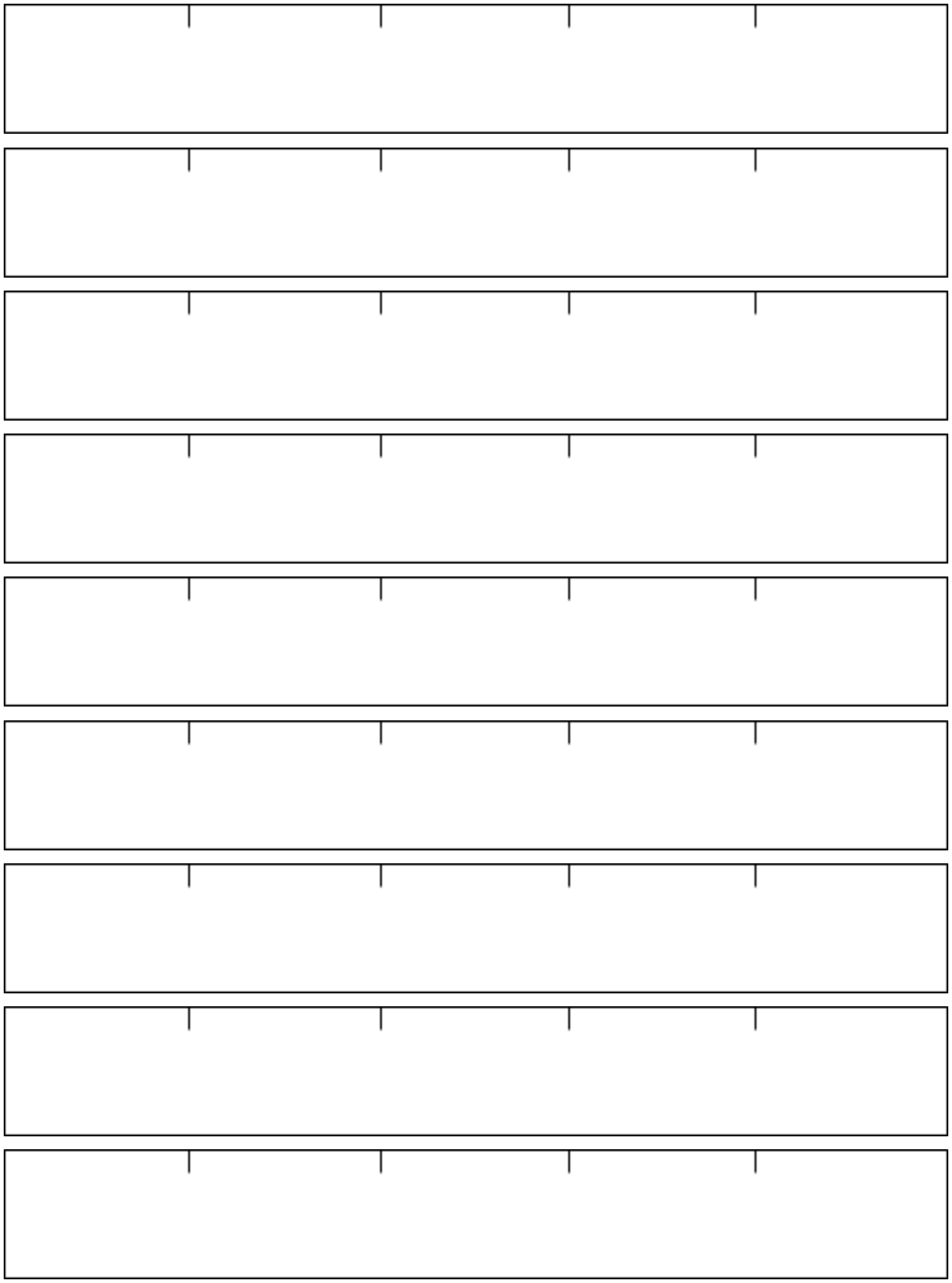
▢ Correct ▢ Not yet	Notes:
Scoring Guidance	A correct answer demonstrates an understanding that ⅔ and ⅔ will be less than 3 pounds total. Student reasoning should include an understanding that two thirds is less than one and so therefore the two bags combined will result in less than 3 pounds. If a student mentally performs the complete addition and converts the improper fraction to a mixed number etc., score as "not yet." This is because if the student is reasoning about the size of the fractions they will quickly see that the sum will be less than 2 and that the complete computation is unnecessary. Ask, "Is there a way to know before doing all of the calculations?"

Part 4 - Show the student the computation cards (C4.1 - C4.4) one at a time. As each card is presented, say, "Read this card." Ensure that they have read the card correctly, then say, "How much is that?"

	Notes: C4.1 ▢ Correct ▢ Not yet C4.2 ▢ Correct ▢ Not yet C4.3 ▢ Correct ▢ Not yet C4.4 ▢ Correct ▢ Not yet
Scoring Guidance	Correct responses are: C4.1: $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$ also accept $\frac{5}{4}$ C4.2: $3 - \frac{1}{4} = 2\frac{3}{4}$ also accept $\frac{11}{4}$ C4.3: $3 \times \frac{1}{8} = \frac{3}{8}$ C4.4: $4 \times \frac{2}{3} = \frac{8}{3}$ also accept $2\frac{2}{3}$

Fractional Reasoning Assessment
FACET 5: Computation

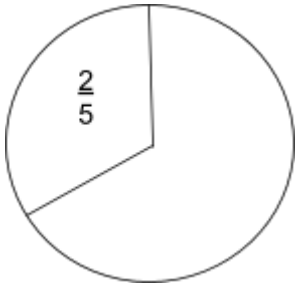
Rectangular strips for Part 1: Prepare one per assessment as necessary.



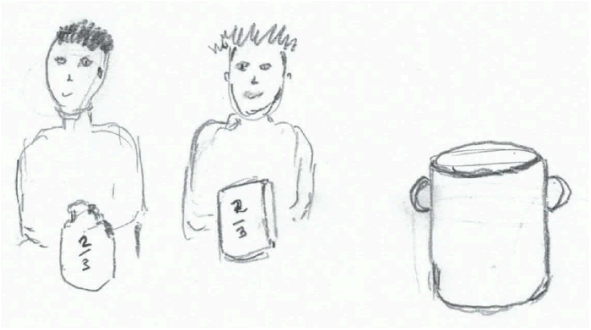
Fractional Reasoning Assessment
FACET 5: Computation

Cards - prepare one set

C2



C3



C4.1

$$\frac{3}{4} + \frac{1}{2}$$

C4.2

$$3 - \frac{1}{4}$$

C4.3

$$3 \times \frac{1}{8}$$

C4.4

$$\frac{2}{3} \times 4$$