Forefront

Fractional Reasoning Screeners 2025

Table of Contents

David Woodward, Forefront Education, 2025

Proficiency with fractions at the end of the elementary school years has been demonstrated to be a strong predictor of success in middle and high school (Siegler, 2012). Understanding fractions is also one of the most ambitious goals of the intermediate grades in mathematics (Braithwaite, 2017; Gabriel, 2013). For many students, learning fractions marks the moment when mathematics becomes completely unintelligible, when number sense becomes number nonsense as teachers instruct students in "copy dot flip," and the use of "butterfly methods." "Ours is not to reason why, just invert and multiply," as the old adage goes.

"As one encounters fractions, mathematical content takes a qualitative leap in sophistication" (Lamon, 2020). Fractions are complex numbers and pose a variety of difficulties for students. For students who are still in the process of conceptualizing place value and who are grappling with multiplicative reasoning, fractions represent a new realm of number ideas. They behave very differently from the whole numbers they have encountered up until now. While whole numbers each have a distinct value, fractions represent the same values in different ways (e.g. 3/4 , 6/8, 9/12, etc.) Meanwhile fractions cannot be understood without the sometimes unintuitive concept of the whole of which they represent a part. To add even further complexity, the whole may be a set (e.g. a package of 12 cookies), making it so that a fraction like ¾ might indicate 9 cookies. Understanding fractions is difficult, but seeking to understand how students understand fractions is fascinating.

Various theoretical frameworks for how students progress in their learning of fractions have been proposed. There are state standards and an evolving research base to help us understand the sequences, stages, progressions, and/or trajectories of learning as students become increasingly competent in their use of fractions (Petit, 2023, Wilkins & Norton, 2018, Fosnot & Dolt, 2002, Battista, 2012, Empson & Levi, 2011, Hackenberg, 2016, Lamon, 2020). The fractional reasoning assessment system presented here acknowledges and embraces that complexity, while primarily serving as an instructionally useful tool for teachers.

Fractional reasoning is a rigorous goal for learning. It requires students to apply their knowledge, communicate their understanding, think flexibly, and achieve computational fluency. When students can reason with fractions, prior knowledge of fractions transfers and becomes applicable to new learning, as they are building on an existing conceptual framework. For students who depend on rote computations with fractions without clear understanding of why those computations work, each new thing to memorize is independent and disconnected.

Defining Fractional Reasoning

Fractional reasoning is a specific type of mathematical reasoning. In their 2011 book *Focus in High School Mathematics*, Dick & Hollebrands define mathematical reasoning as "drawing logical conclusions based on evidence or stated assumptions." The ability to reason is more broadly defined as the ability to use logical, rational, and analytical thought (American Heritage College Dictionary, 1997).

For these assessments, fractional reasoning is being defined as follows.

> *The ability to apply quantitative understandings of fractions to solve problems, and to communicate and justify thinking verbally, symbolically, through gestures, using visual models, and measurement tools.*
>
> *Fractional reasoning does not exclude the use of procedures, but the rote application of procedures without the ability to explain the logic of those procedures does not constitute fractional reasoning.*

How is fractional reasoning revealed?

Students are asked to compare the fractions $\frac{3}{5}$ and $\frac{4}{9}$.

Student 1

"Three fifths is more than four ninths because I know that three fifths is more than one half and four ninths is less than one half. I know this because three is more than half of five, and four is less than half of nine."

This kind of response demonstrates fractional reasoning. It shows an understanding of the relationship of the numerator to the denominator. It also shows an ability to compare to the benchmark fraction of one half.

Student 2

"Three fifths is more than four ninths because I multiply 9 x 3 and that is 27 and I multiply 5 x 4 and that is 20." Teacher: *"How does that work?"* Student: *"I imagine butterfly wings on the fractions."*

In this second example, the student has not communicated fractional reasoning. They have found the correct answer, but have demonstrated they are using whole number thinking, not an understanding of the logic of why three fifths is the larger fraction.

Student 3

"Three fifths is larger. I found common denominators."
Teacher: *"Tell me more. Why does that work?"*

The reasoning will now be revealed in the response of the student. If the student says something like, *"Because common denominators means the pieces are the same size,"* this displays some reasoning. If the student simply explains the procedure, or just says, *"It's cross multiplication,"* or something like that, it shows that their ability to truly **reason** with fractions is still developing.

All three students arrived at the correct answer, but not all three demonstrated a true ability to reason with fractional numbers.

The assessment of reasoning, therefore, must look beyond correct and incorrect answers for evidence of how students understand fractions. This necessitates that teachers pay attention to what students communicate through their words, drawings, and gestures.

Asset-Based Assessment

Asset-based assessments reveal a student's existing knowledge as much as, if not more than, what they still need to learn. If this assessment is successful it will make visible early, and perhaps flawed, understandings of fractions for teachers to be able to build from and connect new learnings.

This means that teachers will need to be attentive and interact with the students throughout the assessment. They should observe, and watch for indications of how students approach the problems. When students are not writing anything, teachers should encourage them to attempt the problems. When students give answers without evidence, teachers should ask them to express their reasoning. When students apply rote procedures, teachers should encourage students to draw pictures and explain their thinking. When a student has done something the teacher does not understand, they should ask the student to explain it. This assessment is designed to help teachers efficiently gather rich, meaningful information in order to learn how their students understand fractions.

When students are unable to demonstrate their understanding clearly, or when the teacher is still curious about a student's thinking, teachers are encouraged to use tasks from the diagnostic assessment. While the screener parts of this assessment are intended to be administered to the whole class, the diagnostic assessment is completed as an individualized interview. The diagnostic interviews are designed to reveal student thinking in more detail, to help teachers identify specific starting points for instruction, and discern assets the student brings with them in order to guide instructional methods and sequences.

What are the Screening and Diagnostic Assessments?

The Fractional Reasoning Screeners are designed to:

- Help educators understand, in general, the fractional reasoning of students and groups of students.
- Help educators identify specific topics for targeted instruction and practice.
- Efficiently and accurately help educators identify students who would benefit from small group instruction and targeted supports.
- Help schools and school systems monitor Tier 1 progress and answer questions like, "Are we improving in our instruction of the ideas aligned with fractional reasoning?" and "Do we see the impact of our efforts and initiatives?"

The Fractional Reasoning Diagnostic Assessments can be used to:

- Probe the understanding of students the teacher was curious about after evaluating the results of the Fractional Reasoning Screeners.
- Reveal inchoate understandings.
- Inform goal setting and planning for instruction by helping teachers to identify prior understandings from which to build new ideas.

Overview

The screening assessments are designed to be administered to whole groups of students. They take approximately 25 - 50 minutes to administer.

The three assessments are:

Early Fractional Reasoning,

which can be used as a 3rd grade post-assessment or 4th grade pre-assessment.

Intermediate Fractional Reasoning,

which can be used as a 4th grade post-assessment or 5th grade pre-assessment.

Advancing Fractional Reasoning,

which can be used as 5th grade post-assessment or a 6th grade pre-assessment.

The first formative is best administered a few weeks before core instruction with fractions begins (or immediately prior if fractions are the first unit of the year.)

The second formative should be administered shortly after units of study related to fractions have been completed.

Scoring the Screener

The analysis, scoring, and data entry of the assessment takes approximately 90 minutes. Detailed rubrics with examples of student work are provided for tasks. Whenever possible, teams of teachers should score the assessments together in order to discuss and consider implications for instruction. Ideally, instructional coaches lead this work. This makes it all more efficient, meaningful, and effective because:

- Teachers are able to discuss and confirm scores with one another.
- Teachers are able to confirm understanding of the rubrics.
- Teachers are able to discuss student reasoning in the moment and notice trends across groups of students.
- Teachers can collaborate with data entry with one person reading scores while the other enters them into the software.

When to Use the Fractional Reasoning Diagnostic

The Fractional Reasoning Diagnostic is a supplemental assessment. The diagnostic assessment is an interview assessment that is conducted with individual students. The amount of time necessary to complete the assessment varies widely from 5 - 35 minutes depending on the student. Students who are early in their development of fractional reasoning tend to take less time, while those who already demonstrate a capacity for more sophisticated reasoning tend to take longer.

The Intended Purposes of the Forefront Fractional Reasoning Screeners

Forefront's Fractional Reasoning Screeners are intended to serve a variety of practical purposes for teachers, schools, and districts.

Screening Assessments

The Fractional Reasoning Screeners help teachers understand students' current understanding in preparation for new learning. Results from the assessment provide helpful formative information for teachers to use in planning instruction and strategic grouping of students. The screeners are designed to help teachers understand student readiness for new learning, along with which facets of fractional reasoning students have developed competency with and which facets might need additional instruction and practice in preparation for new learning. Similarly, these assessments also help to identify students who would benefit from additional instruction in preparation for grade level content. Where there is universal implementation and systematic data collection, the Fractional Reasoning Screeners can be used to monitor systemic improvement.

Formative Assessments

The Fractional Reasoning Screeners are intended to provide rich, formative information and provide insights into student thinking that help teachers identify specific ideas for instruction. They are designed to help teachers understand student thinking so that they can build on that knowledge as they instruct. They are not intended to be used to inform grading. Teachers are encouraged to use the assessment results to provide feedback to students.

Professional Learning

Complementary to the use of the Fractional Reasoning Screeners for formative purposes is the value that these assessments aim to provide for supporting teachers' development of their understanding of how students come to reason with fractions, and the accompanying effective pedagogical practices. This learning can be systematized in a variety of ways: collaborative scoring and analysis of student work, coaching, PLCs, and Data Driven Instruction are some examples.

Recommendation 5 from the IES Practice Guide, Developing Effective Fractions Instruction for Kindergarten Through 8th Grade (NCEE, 2010) centers professional learning as a necessary element for improving outcomes for students. Recommendation 3 states the importance of developing "teacher's ability to assess students' understanding and misunderstanding of fractions." (p.44) The recommendation to "provide teachers with opportunities to analyze and critique student thinking about fractions" (ibid.) is central to the goal of the Fractional Reasoning Screeners.

Progress Monitoring

When any one of the assessments is administered more than once after instruction has taken place, the Fractional Reasoning Screeners can be used to monitor the progress of individuals and groups of students. When the series is used for school-wide and district-wide purposes the assessments can be used to monitor multi-year efforts to improve outcomes with fraction related topics. To this end, the Fractional Reasoning Screeners can be used to help determine the effectiveness of the implementation of instructional programs for teaching fractional reasoning.

Family Communication

The assessments are intended to help inform conversations with the families of students to help them understand how their child is developing in their fractional reasoning and to support them with ideas for fostering that development at home. Forefront® users have access to fully personalized family letters, automatically generated by the software for each student.

Diagnostic Assessments

In addition to the screening tasks, the Fractional Reasoning Screeners provide additional tasks for teachers to utilize when the screening portion of the assessments have not provided sufficient information for guiding their instructional efforts. The diagnostic is intended to be asset-based, that is, the assessments are designed to help teachers identify fractional reasoning and the extent of understanding.

The Forefront Fractional Reasoning Framework

While there is no universally accepted framework for organizing the developmental milestones of fractional reasoning, several frameworks were considered in the writing of these assessments. These theoretical frameworks, some of which are driven by evidence from observing and assessing children, and others which arise from an analysis of the mathematics itself, informed task design as well as grade level alignments. (Wilkins & Norton, 2018, Fosnot & Dolt, 2002, Battista, 2012, Empson & Levi, 2011, Hackenberg, 2016, Petit, et al., 2020, ALCoS, CCSS).

Theoretical frameworks are helpful for understanding student thinking, and they are especially helpful in research. Theoretical frameworks, however, are often misaligned with curricular programs and grade level expectations. In this way, an assessment designed purely around a theoretical framework can be less helpful for informing and guiding classroom practice.

To synthesize the important, evidence-based ideas from the research world with the day-to-day realities of classroom instruction, five levels have been defined which synthesize curricular expectations with developmental progressions and learning trajectories.

The Framework and Standards Alignments

Forefront's Fractional Reasoning Screeners help teachers to identify students' levels of understanding and preparedness for new learning. To help teachers understand where students are in their learning of fractions, the tasks and corresponding results are broken into 5 levels. To make these assessments as practical as possible, there was a deliberate effort to create alignments to curricular expectations as defined by state standards and courses of study in the USA.

The Levels of the Fractional Reasoning Diagnostic Assessment

Level Pre

Students who do not yet demonstrate an understanding commensurate with Level A are considered to be at the Pre level.

Level A - Partitioning (Grades 1-3)

The initial level of fraction conceptualization is the making of fair shares. At this level students are able to form equal sets of discrete objects, partition geometric shapes, and segment lines into equal sized regions. Included in Level A is knowing the words to identify halves and quarters.

Level B - Unit Fractions (Grade 3)

This level includes demonstrating the ability to reason with unit fractions across a variety of contexts. Students at this level can solve problems related to separating and combining fractional parts of wholes in action. Level B includes the idea that unit fractions can be iterated to create a whole.

Level C - Proper Fractions (Grade 3 -4)

Proper fractions are non-unit fractions less than and greater than one. Students who perform at Level C show the ability to demonstrate their understanding of how unit fractions are iterated to form fractions and their relative wholes across a variety of representations and contexts. This same understanding extends to mixed numbers.

Level C also includes the ability to identify and generate equivalent fractions, including conversions of fractions to mixed numbers.

Level D - Operating with Fractions (End of 5th Grade)

The main distinction of Level D is the ability to solve additive and subtractive problems with fractions and to explain their reasoning. Level D of this assessment also includes problems related to the multiplication and division with a whole number and a fraction (unit and proper).

Facets of Fractional Reasoning

Fractional reasoning is multi-faceted. As students deepen their understanding of fractions, they are able to reason with fractions across a variety of modes and contexts. Deep conceptual understanding is evident and grows as students work with a variety of models and metaphors. As Jere Confrey and Alan Malony point out, "An idea unfolds and becomes enriched as one sees various aspects of it and specifically recognizes how it can be refined to make sense of broader related tasks and deeper connections." (2023) No single modality is sufficient for fully demonstrating an ability to reason conceptually with fractions.

Children learn and grow to understand numbers, including fractions, in a variety of ways. There are visual models that students work with to describe units and unit fractions. There are words and syntactic forms. There are problem types, like computational problems and comparison problems. And there are problems that involve fractions of sets, and others that involve the partitioning of discrete wholes.

Petit, Laird, Ebby & Marsden (2023) suggest that students' recognition of fractions as numbers, in contrast to whole number reasoning, can be revealed when students:

- "locate fractions on a number line
- compare fractions
- identify fractional parts of wholes
- estimate the magnitude of fractions
- operate with fractions"

The assessment system presented here uses a similar set of ideas for distinguishing fractional reasoning.

6 Facets of the Forefront Fractional Reasoning Screeners

The Fractional Reasoning Screeners probe the understanding of fractions across 6 facets. These same facets are also seen in most common instructional materials and are explicitly called out in many state standards and curricula.

Facet 1: Words and Symbols

This includes being able to name fractional parts as well as being able to read and write fractions. This includes questions like:

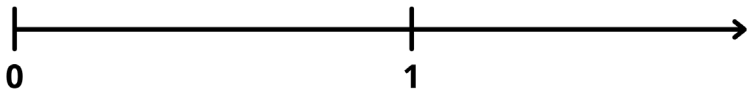
What part of the rectangle is shaded? (Level A task)

Facet 2: Shapes

This facet includes the ability to connect symbolic representations of fractions to reasoning with 2 and 3-dimensional shapes: rectangles, circles, rectangular prisms, etc. This includes tasks like, "Here is a rectangle. Show $\frac{3}{5}$ of this rectangle." (Level C task)

Facet 3: Lines

This facet includes the ability to connect fraction numerals to number lines.
This includes tasks like: Where does ⅔ belong on this number line? (Level C task)



Facet 4: Sets

Sets are discrete, not easily separable items. Pennies, buttons, candies, dots, etc. This facet includes questions like, "I have a package of 12 candies. I want to give $\frac{1}{3}$ to my friend. How many should I give them?" (Level B task)

Facet 5: Magnitude, Comparison, and Equivalence

This facet of reasoning, at the most basic level, includes situations like, "Julieta and Jack each have crackers. Jack breaks his cracker in half and says, 'Look, I have more! I have 2, you only have 1.' Julieta says, 'No you don't!' Who do you agree with and why?" (Level A task)

Facet 5 also includes classic comparison problems using <, >, = and problems that require finding a common denominator. The assessment of reasoning when comparing necessitates questions that ask how a student knows and examines whether they can justify their reasoning for why one fraction is greater than another, or why they are equivalent.

Facet 6: Computation

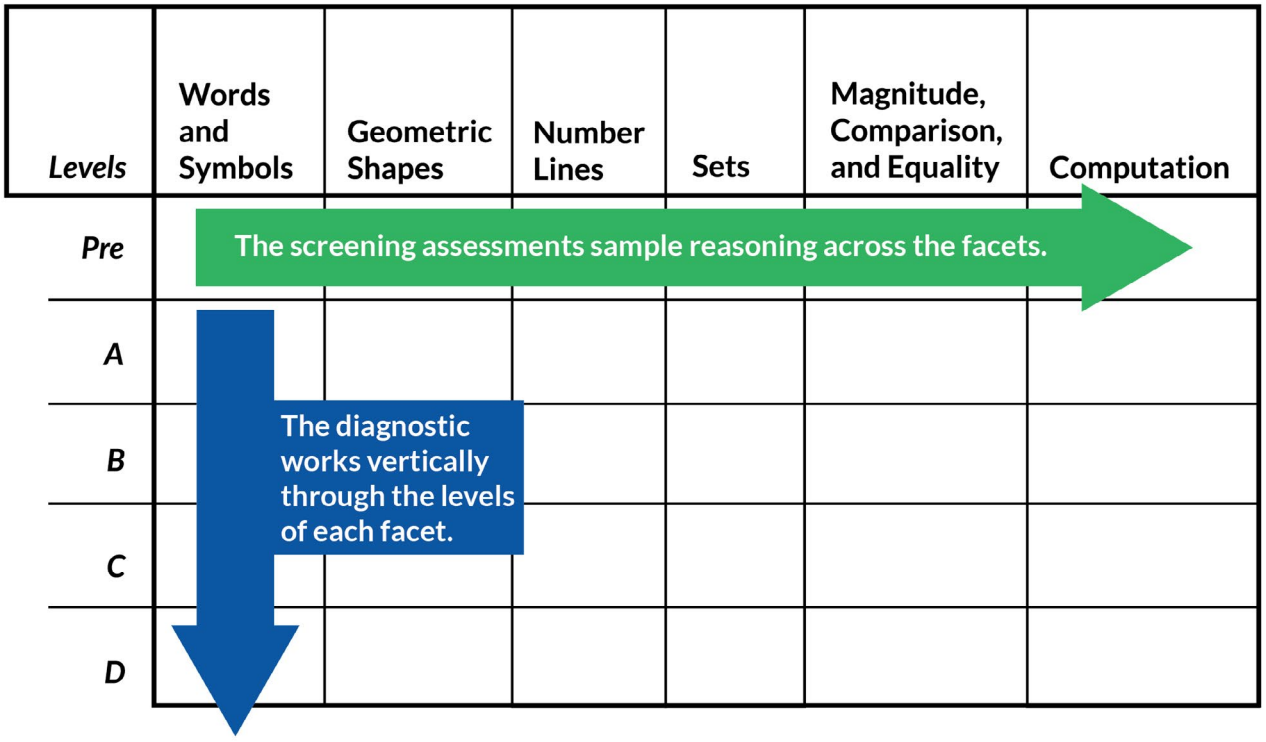
Computation includes the combining, separating, or interaction of fractions. At the most basic levels, this includes recognizing which shape is complementary to another for the creation of a geometric figure. At the highest levels of this assessment series, computation involves the addition and subtraction of mixed numbers, the multiplication of whole numbers by fractions, and fractions divided by whole numbers.

The ability to apply reasoning (in contrast with whole number thinking and procedures) to compute with fractions is assessed using contextualized problems. This encourages students to apply their visual thinking and to be less likely to rotely apply a known procedure.

The Overall Structure

Each of the facets is used to elicit thinking across the different Levels, progressing from less complex to more complex as the Levels increase.

The Fractional Reasoning Screeners work students across a series of tasks approximately at the same level, depending on their grade level. The tasks provide opportunities for the students to express their fractional reasoning abilities across each of the 6 facets described above. Therefore, the screening assessments sample reasoning across each of the facets, moving horizontally, from left to right, across the chart on the next page.



While a screening assessment cannot thoroughly assess proficiency across the full spectrum of this framework, the tasks help teachers to understand thinking across each of the facets, and hone in on these different aspects of fractional reasoning.

In contrast to the screening assessments, the Fractional Reasoning Diagnostic is designed to work vertically across each of the facets of the assessment. That is, teachers are encouraged to choose a facet of fractional reasoning and probe student thinking with questions across each facet.

Progressive Formalization and the Fractional Reasoning Screeners

Progressive formalization is an idea that has come from the Freudenthal Institute and Realistic Math Education (Van Reeuwijk, M., 2001, Gravemeijer, K., 2003, Brendefer, J., 2021). Progressive formalization proposes that mathematical ideas, when presented in meaningful contexts, are more accessible, and enable students to engage meaningfully with the problem. When tasks are presented informally, students are more likely to apply their intuitions and reasoning to solve tasks than they would be with more formally presented tasks.

The tasks of the Fractional Reasoning Screeners and Diagnostic assessments can be put into three problem types defined by the ideas of progressive formalization: informal, preformal, and formal (Van Reeuwijk, 2001, Gravemeijer & van Galen, 2003, Webb, D. C.2008, Bredefer & Strother, 2021).

Informal, Preformal, and Formal

Informal

Here are two whole crackers. If you break each of these crackers into thirds, how many pieces will you have?

Preformal

Divide each of these rectangles into thirds. How many thirds did you make?

Formal

Solve $2 \div \frac{1}{3}$

Division of a whole number by a unit fraction, when presented formally, is considered 5th grade content according to the CCSS and most state standards. But, the informal presentation makes this problem accessible to even some 1st and 2nd grade students. Susan Empson and Linda Levin, in their book Extending Children's Mathematics: Fractions and Decimals (2011), illustrate in detail how well-designed contexts can make problems accessible to students with a wide range of understanding of fractions.

Contexts and visual models have been used deliberately throughout the Forefront Fractional Reasoning Screeners. This helps teachers to recognize students' ability to engage with the topics at a variety of levels of formality. Students who might be unable to clearly explain their reasoning with more formally presented tasks are able to reason with similarly structured problems at less formal levels.

Informal Tasks are contextualized and easily accessible. The context provides opportunities for the students to act out and/or easily imagine the situation. Think of a rectangle. To make this less formal, think of it as a graham cracker. To make it even less formal, provide a paper rectangle for the student to work with. To make it even less formal, use an actual graham cracker. Informal tasks invite intuition and action. Informal tasks typically include no symbols.

Preformal Tasks use physical and visual models that are teacher-created. They are a step toward the abstraction of an idea. Preformal settings are typically adult-invented instructional models and materials like ten-frames, base-ten blocks, abacuses, and fraction pieces. In these assessments, rectangles, circles, and number lines are considered preformal models.

Formal Tasks are generally presented symbolically without context.

While progressive formalization is often thought of as a one-way progression from informal to formal, this is a misconception and with fractions this is most definitely not the case. Quite often we will encounter students who are able to demonstrate some fluency with tasks presented formally (e.g. ¾ + ⅔), yet are unable to match that same problem to a pre-formal representation, or are often even less able to imagine an informal application for such a problem. A student who fully demonstrates fractional reasoning can demonstrate their understanding in informal, preformal, and formal ways.

Conclusion

Fractional reasoning is important for all students to develop. The ability to reason with fractions serves as a conceptual springboard for learning algebra and higher mathematics in both middle and high school. As important as this learning is, it is difficult to achieve and to assess. These assessments are designed to inform teacher understanding of their students, and of the topic itself in order to inform and improve instruction and therefore learning outcomes for all students.

Battista, M. T. (2012) *Cognition-Based Assessment & Teaching of Fractions: Building on Student's Reasoning*. Heinemann. Portsmouth, NH.

Braithwaite, D. W., Pyke, A. A., & Siegler, R. S. (2017). *A computational model of fraction arithmetic*. Psychological Review, 124(5), 603-625. doi: 10.1037/rev0000072.

Bredefer, J. & Strother, S. (2021) *DEVELOPING MATHEMATICAL THINKING FRAMEWORK AND CLASSROOM STRUCTURE*. DMT Institute, Boise, ID (retrieved from https://www.nmlegis.gov/handouts/ALESC%20042722%20Item%204%202a%20-%20DMT%20Framework.pdf)

Confrey, J. & Maloney, A. (2023) Learning Trajectories and Equity: Making a Strong Link Stronger. Retrieved from: https://cadrek12.org/resources/blogs/learning-trajectories-and-equity-making-strong-link-stronger

Dick & Hollebrands (2011) *Focus in High School Mathematics*. National Council of Teachers of Mathematics. Reston, VA.

Empson, S. B., Levi, L. (2011) *Extending Children's Mathematics: Fractions and Decimals*. Heinemann. Portsmouth, NH.

Fosnot, C. T., Dolk, M. (2002) *Young Mathematicians At Work: Constructing Fractions, Decimals, and Percents*. Heinemann. Portsmouth, NH.

Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B. (2013). *A componential view of children's difficulties in learning fractions*. Front Psychol. 2013 Oct 10;4:715. doi: 10.3389/fpsyg.2013.00715. PMID: 24133471; PMCID: PMC3794363.

Gravemeijer, K., & van Galen, F. (2003). *Facts and algorithms as products of students' mathematical activity*. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), A research companion to principles and standards for school mathematics (pp. 114-122). Reston, VA: NCTM.

Hackenberg, A. J., Norton, A., Wright, R. J. (2016) *Developing Fractions Knowledge*. Sage. Thousand Oaks, CA.

Lamon, S. J. (2020) *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers* (4th ed). Routledge. New York, NY.

Petit, M.M., Laird, R.E., Ebby, C.B. & Marsden, E. L. (2023) *A Focus on Fractions: Bringing Mathematics Education Research to the Classroom (3rd Ed)*. Routledge. New York, NY.

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide (NCEE #20120-4039)*. Washington, DC: National Center for Education Evaluation Sciences, U.S. Department of Education. Retrieved from whatworks.ed.gov/publications/practiceguides.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early Predictors of High School Mathematics Achievement. Psychological Science, 23(7), 691-697. https://doi.org/10.1177/0956797612440101

Steffe, L. J., John, O. (2010) *Children's Fractional Knowledge*. Springer. New York, NY.

Van Reeuwijk, M. (2001). *From informal to formal, progressive formalization an example on "solving systems of equations"*. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), The future of teaching and learning of algebra: The 12th ICMI study conference (pp. 613–620). Melbourne, Australia. Retrieved from http://repository.unimelb.edu.au/10187/2812

Webb, D. C., Boswinkel, N., & Dekker, T. (2008). *Beneath the Tip of the Iceberg: Using Representations to Support Student Understanding*. Mathematics Teaching in the Middle School, 14(2), 110-113.

Wellberg, S., Briggs, D. C., Sanford, S.R. (2023) *Big ideas in the Understanding of Fractions: A Learning Progression*. A Center for Assessment, Design, Research and Evaluation (CADRE) Working Paper, University of Colorado Boulder. Retrieved from: https://live-ucb-cadre.pantheonsite.io/sites/default/files/attached-files/wbs_cadre_report_fractions76.pdf

Wilkins, J.L.M., Norton, A. (2018) *Learning progression toward a measurement concept of fractions*. IJ STEM Ed 5, 27 (2018). https://doi.org/10.1186/s40594-018-0119-2.

Instructions for the Administration of the Early Fractional Reasoning Screener

This assessment is intended to be administered to the whole class at once. A script is provided with detailed instructions, look-fors, and approximate timings.

The entire test will take 25-40 minutes depending on your students. There will likely be students who are unable to complete some of the tasks during the time provided. Plan to provide the time at the end of the assessment or at a later time for those students.

This assessment is different from state or other formal tests. Teachers are expected to engage with students, to re-read questions as necessary, answer questions, and encourage students to elaborate on their thinking. The purpose of the assessment is to gather as much evidence of student thinking as possible, and teachers are critical in this process.

When should this assessment be used? This assessment has been designed to be used at one of the three times below.

3rd Grade

- Administer in the midst of fractions instruction as a formative assessment.
- Administer the assessment towards the end of instruction related to fractions to identify topics for re-teaching, additional practice, and targeted instruction.

4th Grade

- Use this assessment a few weeks before or early in the instruction of fractions in 4th grade as a readiness assessment and to identify areas for pre-teaching before fraction units of instruction begin.

Preparation

- Make copies of the assessment for all students.
- Have pencils with erasers (not pens) for all students.
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the video: There is a video that accompanies this assessment for question 6. Have the video cued before you begin the assessment. (Access the link here or here: https://bit.ly/FRS-Early6.)

Assessment Administration

Please follow the script as written and have a clock or timer ready. The times are flexible and are provided as guidance in order to keep the assessment moving at a reasonable pace. It will also ensure that the assessment remains efficient. Students should not be aware of the timing of the tasks. This is not a timed test.

You may re-read questions.

Answer questions to ensure students understand the task. Refrain from answering questions that would influence student thinking or provide them clues to how to solve the tasks.

Active Proctoring

Teachers are encouraged to move about the room and to interact with the students.

- When students are not engaging, encourage them and re-read the question for them personally to ensure understanding.
- When you see student work that leaves you uncertain about the reasoning, ask questions and encourage them to write, draw, use number lines, or demonstrate their thinking in other ways.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning. See the script for look-fors and additional prompts to consider during the assessment.

Data Collection for Forefront® Users

Forefront has been configured to support data collection for Forefront® clients. Questions can be sent to support@forefront.education. Your account manager will support your district in account configuration and rostering to enable data entry by teachers.

Observations

Look-fors during assessment are called out in the script and in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring

Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the student to ensure the accuracy of the scoring and the analysis.

Accommodations

Any special accommodations which would normally be given for individual students should also be given for this assessment.

Pacing Guidelines

Suggested timings are provided for the tasks, however this is **not** a timed test. Give students enough time for each task so that it doesn't feel rushed, but also keep the test moving at a pace that keeps the students engaged and focused. Some students will likely need extra time after you have finished all the tasks to complete their work. It is recommended that teachers have a clock visible while giving the test in order to help pace the assessment, but students should not be made aware of this.

Script for the Administration of the Early Fractional Reasoning Screener

Before You Begin the Assessment

- Read these instructions completely to familiarize yourself with the assessment.
- Prepare your classroom to minimize opportunities for students to see each other's work.
- Prepare to project the video to be shown for Task 6 (https://bit.ly/FRS-Early6).

How to Pace the Assessment

This assessment should take 35-50 minutes. To keep the students working at a good pace, timings are provided for each question. Prepare a clock or timer before you start, but be discreet with it. This is not a timed test, but the timings are provided as guidelines. Teachers are encouraged to use their best judgment as they lead the students through the assessment. Once almost all of the students have completed each task, continue to the next task and let students know that they will be provided with time to return to unfinished tasks later if necessary.

The Script

Read all of the bolded text.

Teacher: **Today we are going to solve some problems to help me to understand how you understand fractions. The results of this assessment will not be part of your grade. This is only to help me to see how you understand fractions and to help me teach you better. For that reason it is important to always explain your answers clearly. Use words, drawings, number lines, and numbers to show your thinking.**

Please keep your eyes focused on your own paper. It is important that I get to know your own thinking about these ideas so that I can support you better. (Some teachers may want to spread students out a bit, or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please try not to work ahead, since these questions are a little different than other assessments. The last question will include a video that we will watch together.

If at any time you have a question, please raise your hand. If you need me to repeat a question, let me know. Are there any questions before we get started? *(Answer questions as necessary.)*

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

Continue when all of the students are ready.

EARLY

Task 1

Teacher: **For the first question I will read some numbers aloud. Write the number, not words, in each of the boxes.**

In box A write the number: one half.
In box B write the number: three eights.
In box C write the number: five quarters, or five fourths.

Look-Fors

If you see students writing words (like "one half"), redirect them to write the number as a fraction.

Pause briefly after you read each of the numbers for Task 1 to give students the opportunity to write their numbers. Continue at a reasonable pace. Repeat numbers as necessary to ensure every student has heard.

Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Task 2, Part A

Teacher: **Number two has two parts. Part A says, "Shade three-fourths of each of the shapes below. Shade three fourths of the rectangle and three fourths of the circle."**

Walk around the room to ensure that students get started. Pause to allow students to complete part A, then continue.

Task 2, Part B

Teacher: **Find Part B and put your finger on it.** (Look for students to find Part B.) **Part B says, "Here is a gray rectangle. In the space below, draw a rectangle that is four sixths the size of the gray rectangle." You may make marks on the gray rectangle to help you. Be sure to show your thinking.**

Walk the room to answer questions, check that students are getting started, and to ask students to clarify their thinking.

Questions to ask individual students as necessary:

- **Show me how you know yours is the right size.**
- For students who are hesitant to respond **- How big do you guess it would be?**
- For those who solve it quickly and accurately **- Are there other ways to make a rectangle four sixths the size?**

When most students are done or around 3 minutes pass, say, **We are going to continue to the next task. If you have not finished, don't worry, I will give you time to finish later.**

Look-Fors

- Are students gesturing and measuring with their fingers?
- Are students making marks on the gray rectangle?
- Are students partitioning the gray rectangle?

Pacing Guideline

about 3 minutes

Note

Do not provide, or allow students to use rulers. If a student asks to use a ruler or some other length measurement device, say something like, **That's not a bad idea, but just estimate for now. It does not need to be perfect.**

Task 3, Part A

Teacher: **Turn the page and find number 3. This question also has two parts. Part A says, "Here is a number line with the numbers 0 and 1. Put the numbers one half and one fourth where they belong on the number line."**

For students who respond quickly and accurately, ask

- **Where would one third go?**

Pacing Guideline

about 1-2 minutes

Task 3, Part B

Teacher: **Find part B. Part B says, "Here are some more fractions and a new number line. The number line below has the numbers zero, one, and two. Put the fractions onto the number line."** (Do not read the fractions.) **"Make a mark where each number belongs, and write the number by the mark."**

Circulate the room to answer questions and ensure that students are clearly marking their number lines and writing the fractions to indicate which fraction goes with which mark.

Questions to ask individual students as necessary:

- **How do you know it goes there?**
- For those who quickly show proficiency - **Are there other numbers you could put on the line?**

After 3 - 5 minutes, or as students finish, say, **We are going on to the next question now. If you have not finished, don't worry, I will give you time to work some more later.**

Look-Fors

- Are students making additional marks to support their thinking?
- Are students using their fingers to measure?

Pacing Guideline

about 3-5 minutes, move on if done sooner

Note

Do not provide, or allow students to use rulers. If a student asks to use a ruler or some other length measurement device, say something like, **That's not a bad idea, but just estimate for now. It does not need to be perfect.**

Task 4, Part A

Teacher: **Number four also has two parts. Part A says, "The rectangle with circles below represents a small pack of candies. I want to give one third of the pack of candies to a friend. Show how many of the candies I want to give my friend."**

Write the number of candies on the line. Circle the candies, or draw them in the space to show your thinking.

Look-Fors

- Some students say they want to be fair and give half to their friend. If you see that happening, check to see if the student understands the question. Try to do this without leading the student, just ensure understanding.
- Are students only putting an answer? Ask them to show how they know.

Pacing Guideline

about 1-2 minutes

Task 4, Part B

Teacher: **Find Part B. Part B says, "The rectangle with circles below represents a pack of 12 candies. I want to give three fourths of the candies to a friend. Show how many candies I want to give my friend. Write the number of candies on the line."**

Look-Fors

- If a student writes numeric answers quickly, encourage them to make a drawing or explain how they know.

Pacing Guideline

about 3 minutes

Task 5

Teacher: **Let's move on to Number 5. If you haven't finished number 4, don't worry, we can come back and finish it later. "Use the greater than, equal to, and less than symbols to compare each pair of fractions. Make drawings to explain your answers."**

Be sure to explain your thinking using number lines, drawings, or sentences. Circulate to remind students to explain their reasoning for each comparison.

Pacing Guideline

about 5 minutes

Look-Fors

- Ask students who use only numbers (i.e. they find common denominators) to show their thinking with pictures, number lines, or words.
- If students use only shapes to compare the two, ask if they have another way to know that they are correct.

Note

You may clarify for students the use of the > and < symbols. If there is any uncertainty, you may ask the student to circle the larger fraction. The goal of this assessment is not to determine whether they can use the symbols accurately, it is to see how students reason about the relative size of the fractions.

Task 6

Teacher: **We are going to continue now to number 6. This question includes a short video. First I will read it, and then we will watch the video that goes with the question. I will tell you when to start after we watch the video. "A circle is divided into three equal parts. One part is cut off and removed. How much of the circle remains? Write a fraction and draw to show your answer."**

Pacing Guideline

about 5 minutes

Please show your thinking in the space provided and write a fraction on the line.
Play the video. (Access with the QR code or this link or here: https://bit.ly/FRS-Early6.)

Write a fraction and draw to show how much of the circle remains.

Conclude the assessment. As you collect the papers, check to see if there are unfinished problems and either keep them for finishing later, or ask the student to complete them. For students who need more time, provide it now, or later.

EARLY

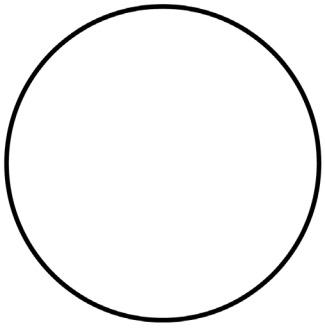
Name: _____ **Date:** _____

Show what you know about fractions on this assessment -- showing your thinking is as important as getting the answer.

1. Write the numbers in each of the boxes as your teacher reads them.

A	B	C

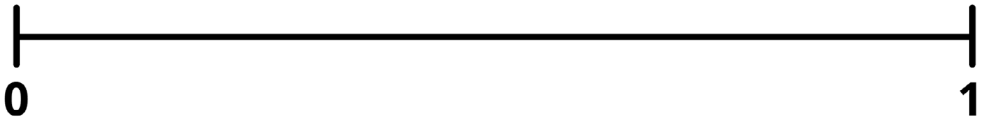
2. Part A: Shade $\frac{3}{4}$ of each of the shapes below.



2. Part B: Here is a gray rectangle.

In the space below, draw a rectangle that is $\frac{4}{6}$ the size of the gray rectangle.

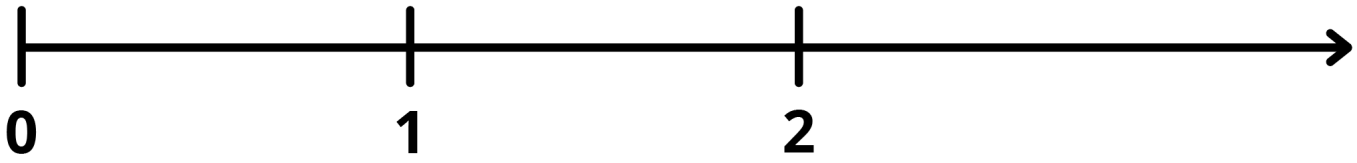
3. Part A: Here is a number line with the numbers 0 and 1.
Put the numbers $\frac{1}{2}$ and $\frac{1}{4}$ where they belong on the number line.



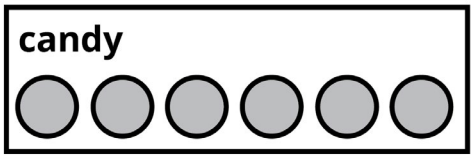
3. Part B: Here are some more fractions and a new number line.
The number line below has the numbers 0, 1, and 2.
Put the fractions onto the number line.

$\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{1}$ $\frac{4}{3}$

Make a mark where each number belongs, and write the number by the mark.

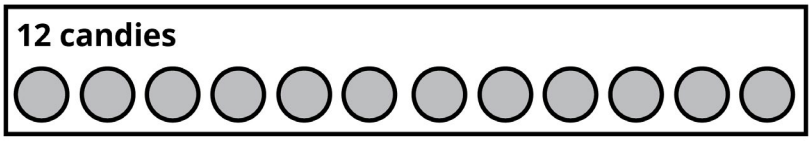


4. Part A: The rectangle with circles below represents a small pack of candies.
I want to give $\frac{1}{3}$ of the pack to a friend. Show how many of the candies I want to give my friend.



_____ candies

4. Part B: The rectangle with circles below represents a pack of 12 candies.
I want to give $\frac{3}{4}$ of the pack to a friend. Show how many of the candies I want to give my friend. Write the number of candies on the line.



_____ candies

5. Use >, =, < to compare each pair of fractions. Make drawings to explain your answers.

$\frac{1}{3}$ $\frac{1}{4}$ $\frac{3}{8}$ $\frac{3}{4}$ $\frac{3}{3}$ $\frac{5}{5}$

6. Video: A circle is divided into 3 equal parts. One part is cut off and removed. How much of the circle remains? Write a fraction and draw to show your answer.

Draw in the space below.

_____

fraction

Forefront Data Entry

Enter results as instructed in each of the questions.

Standards Alignments

At this time, these questions are simply aligned with the 3rd grade domain of fractions - e.g. 3.NF. Specific alignments for different states will be done and configured in Forefront over time.

Fractional Reasoning Lens Facets

Alignments to the Fractional Reasoning Lens are listed with each of the tasks. For more on the lens, see the introduction of this document.

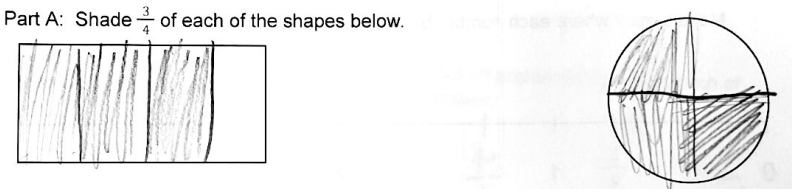
Observations

For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These may be taken into consideration during the scoring.

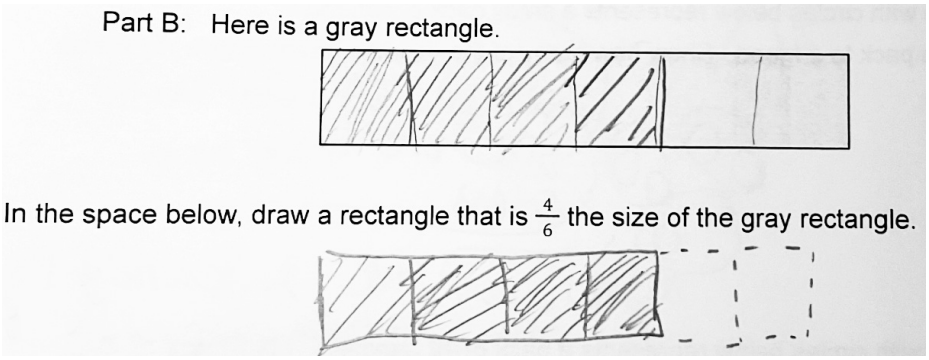
Task 1: Writing Fractions from Dictation

Facet	FR.3.WS (Words and Symbols)
Performance Levels	• Meeting (3 points) • Approaching (2 points) • Not yet (< 2 points)
Answer Key	A. ½ B. ⅜ C. ⅝ One point for each correctly written fraction (enter points in Forefront).
Clarifications	Credit should be given if the number is legible. While students should be encouraged to write the numerals vertically, give points if they have written them like this: ½.
Readiness Expectation	Students should be able to write unit and proper fractions from dictation.

Task 2, Part A: Shade 3/4 of Geometric Shapes

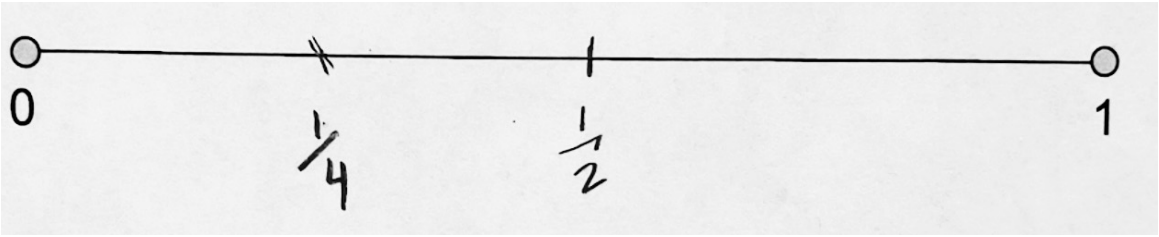
Facet	FR.3.Shapes (Shapes)
Performance Levels	• Both accurate (2 points) • Approaching (1 point) • Not yet (0 points)
Answer Key	4 regions should be approximately equal in size, and three of the sections should be shaded. One point for each correctly shaded shape. 
Readiness Expectation	This task is looking for 2 elements: • Students should recognize that the shapes need to be partitioned into 4 equal parts. • Three of those sections need to be shaded to represent ¾. If the student has split the circle using parallel lines, consider it incorrect. Conceptually, the student might know that the sizes of the partitions need to be identical (or they might not). Even if the student has indeed intended to make fair shares, this should be considered an important point for clarification, since it is nearly impossible to slice a circle with parallel lines and arrive at equal-sized regions.  It should be noted that this problem suggests a very basic understanding of fractions, and while students should be able to readily complete this task, being able to complete this task is not a strong indicator of fractional reasoning. This is because for some students this problem has become so rote, that they simply know to cut into 4 and shade 3, essentially working through the process with only whole number thinking.

Task 2, Part B: Show 4/6 of a Rectangle

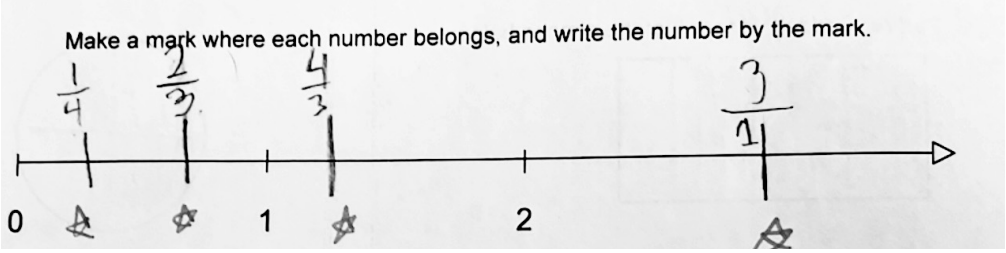
Facet	FR.3.Shapes (Shapes)
Performance Levels	• Shows reasoning (1 point) • Not yet (0 points)
Answer Key	Student accurately draws a rectangle that is 4/6 the size of the gray rectangle and demonstrates partitioning and iterating. 
Clarification	Student demonstrates an ability to reason that the gray rectangle needs to be divided into 6 equal parts, and then a rectangle the size of 4 of those parts should be drawn in the space below. If a student has hand drawn a rectangle approximately the correct size without any indication of how that was done, encourage the student to explain their answer (either during or after the assessment). Students should apply partitioning (breaking the gray rectangle into 6 equal sized pieces), and then iterating (repeating the sixths) to create the rectangle which is approximately accurate in size. What if they show 2/3? 4/6 was chosen as the fraction for this question for its potential for revealing student understandings of equivalence. If a student draws a rectangle that shows that it is 2/3 the size of the gray rectangle, take note. In the above example, the student has shown how they partitioned the gray rectangle into 6 equal parts and reproduced the rectangle below. Notice how the student also shows the two-sixths that have been removed.

Readiness Expectation	This problem takes two steps toward complexity to see whether students have fully understood the idea of partitioning and iterating. By using 6 for the denominator it forces the student to think carefully about where those partitions need to happen, and perhaps adjust some. The second complexifying aspect of the question is asking the student to reproduce the fractional piece separate from the original rectangle. This is a step toward students understanding fractional parts of a whole while not destroying the original whole. This is called "disembedding" (Hackenberg, Norton & Wright (2016).

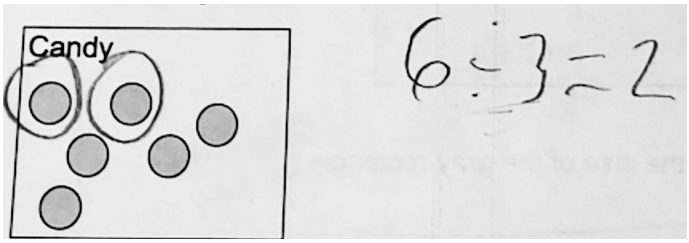
Task 3, Part A: Half and Fourth on Number Line

Facet	FR.3.NL (Number Lines)
Performance Levels	• Shows reasoning: Student shows an understanding of the placement of unit fractions on a number line that goes from 0-1. (2 points) • Not yet: Student is still developing an understanding of how to place unit fractions on a simple 0-1 number line. (<2 points)
Answer Key	Give one point for each correct response. ½ should be approximately half way between 0 and 1, and ¼ should be approximately half way between 0 and ½. 
Readiness Expectation	This task represents a starting point for fractions on number lines. Students who are unable to accurately place ½ and ¼ on a number line should be provided with targeted instruction. End of year expectations for fractions in 3rd grade is that students can place fractions, including fractions greater than one (improper fractions) on number lines. For students who finish quickly, and for more information, consider asking where the number ⅓, and perhaps ⅕, should go on the line. While this is not scored on this rubric, these questions can reveal more about students' understanding of unit fractions on number lines and thus provide important formative information.

Task 3, Part B: Fractions on a Number Line

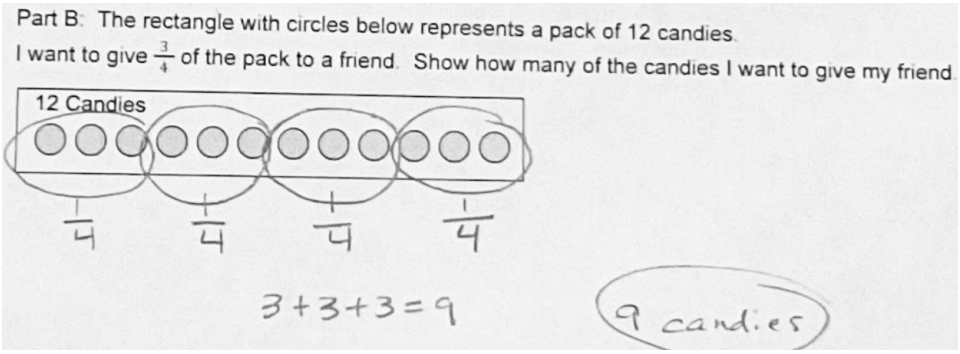
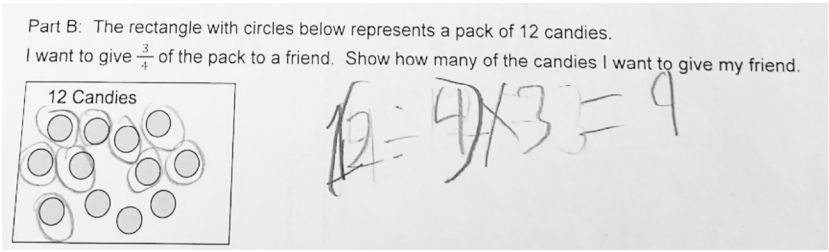
Facet	FR.3.NL (Number Lines)
Performance Levels	• Shows reasoning (4 points) • Approaching (3 points) • Not yet (<3 points)
Answer Key	Give one point for each correctly placed fraction: • ¼ is left of center between 0 and 1. • 2/3 is right of center between 0 and 1. • 4/3 is left of center between 1 and 2. • 3/1 is placed where the number 3 would be. 
Readiness Expectation	By the end of 3rd grade, students should be able to represent fractions on number lines that extend beyond 1. The importance of the line extending beyond 1 is that it forces the student to first consider the whole before partitioning it. The ability to accurately place fractions on lines demonstrates their ability to reason about the magnitude of the fraction relative to the unit. To place the numbers accurately, watch for students who are measuring with their fingers or pencils to divide the units into fractions. Students may make additional marks on the line to support their thinking. This is good evidence of their understanding that they need to break the whole numbers into the appropriate number of pieces and then, if necessary, iterate those to correctly place the fractions on the line. This question can reveal students who are still struggling with thinking about fractions as numbers, distinct from whole numbers. For example, many students imagine ⅔ belongs somewhere between 2 and 3. Note: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long.

Task 4, Part A: One Third of a Set of Six

Facet	FR.3.Sets (Sets)
Performance Levels	• Correctly answers 2 (1 point) • Not yet (0 points)
Answer Key	Give 1 point if the student answers 2.
Clarification	While reasoning is not required for a one-point answer, reasoning gives a window into student thinking. If a student simply answers two, and the thinking is not clear, ask the student, "Can you tell me how you know?" Encourage students to write or draw an explanation of their answer. Verbal explanations are acceptable. 
Readiness Expectation	Students recognize that in order to find ⅓ of a set, the set needs to be partitioned into 3 equal sized sets. While the ability to find a fraction of a set is not clearly defined in most state standards, the idea of forming equal groups is associated directly with ideas of whole number division. Students should be able to demonstrate that fractions of sets can be found by partitioning the larger set into smaller groups of equal size. Many students will relate this to the work with whole number multiplication and division that is central to the work of 3rd grade. That is good.

Task 4, Part B: 3/4 of a Set of 12

Facet	FR.3.Sets (Sets)
Performance Levels	• Student answers 9 candies (1 point) • Not Yet (0 points)
Answer Key	Give 1 point if the student answers 9.

Clarification	While reasoning is not required for a one-point answer, reasoning gives a window into student thinking. Reasoning on this task is visible when the student has clearly found the unit fraction (¼) is equivalent to 3 candies. Further, these students show that three 1/4s are represented by 9 candies.  
Readiness Expectation	In order to find ¾ of the pack, students need to recognize that the complete set needs to be divided into 4 equal groups to identify the unit fraction, and then iterate that group 3 times.

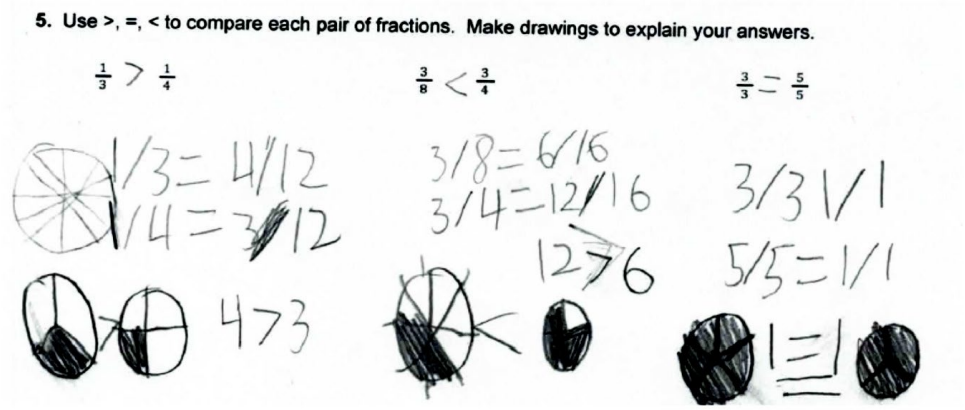
Task 5: Comparisons

Facet	FR.3.MCE (Magnitude, Comparisons & Equivalence)
Performance Levels	• Demonstrate the use of reasoning about size to compare basic fractions (3 points) • Approaching (2 points) • Not yet (<2 points)
Answer Key	Give one point for each correct fraction comparison and accompanying reasoning. Read the clarifications to better understand appropriate reasoning.

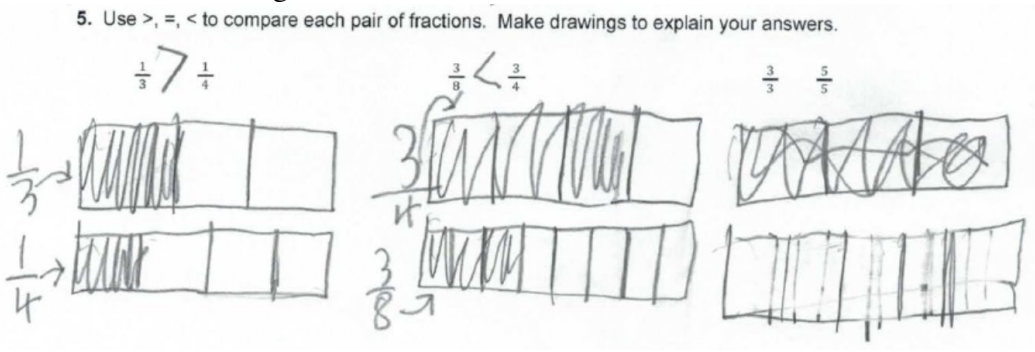
Clarification

- <u>It is not enough to simply put a comparison symbol.</u> If a student has only answered with a symbol, ask them to explain their answer with pictures, number lines, words, or numbers.
- Students should explain their reasoning about the size of the fractions in order to compare them. Score their reasoning. If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<,>,=) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol and give a point for showing reasoning about size.
- If the student is unable to reason about the size of the fractions, but instead applies some sort of procedure to correctly compare the fractions, ask the student if they have another way to show how they know their answer is correct. For example, if a student says that they know that ⅓ is greater than ¼ because the denominator is smaller, ask them to make a drawing to show how that works.
- If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, or other means to explain their reasoning about the size of the fractions, <u>DO NOT give a point, even if the comparison is correct.</u>

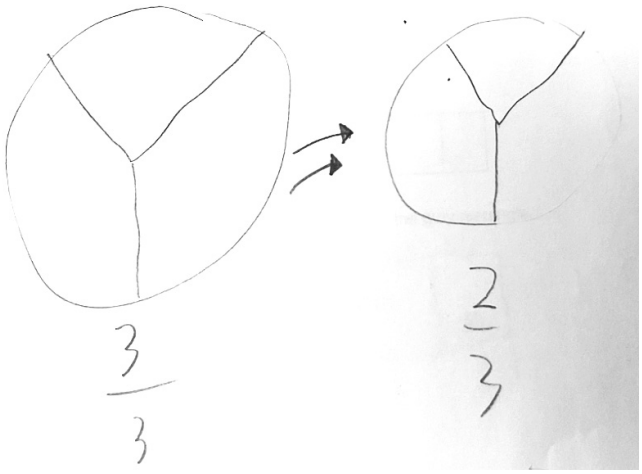
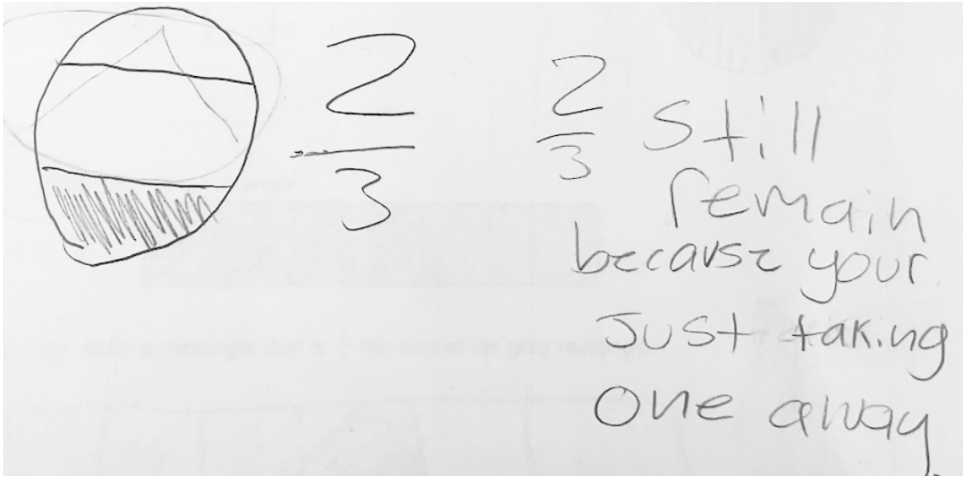
<u>The sample below represents a score of 3.</u> Notice that each comparison is accompanied by a drawing and that the numerical equivalents also make sense.



<u>The sample below should be given a score of 2.</u> The first two comparisons are correct and justified with a drawing.



Task 6: 1 - 1/3 of a Circle

Facet	FR.3.C (Computation)
Performance Levels	• (1 point) Student shows the ability to reason about the circle and the missing piece to know that ⅔ of the whole circle is what remains. • Not yet (0 points)
Answer Key	This question requires that the student answers ⅔, and draws an image that shows approximately ⅔ of a circle. What we are looking for here is the ability to visualize the fractional amount and to tie that to the symbol. A correct answer shows the number ⅔ as the amount remaining and a drawing that approximates ⅓ of a circle being removed from a whole circle. 
Clarification	The sample below should also be scored as "shows reasoning." (1). Although the partitions are not even in the drawing, and do not reflect exactly what was in the video, as it relates to the problem solving and arriving at the correct answer, the student has demonstrated the reasoning that this question is assessing. Nevertheless, attention should be paid during instruction to explain why the drawing is not accurate. 

Instructions for the Administration of the Intermediate Fractional Reasoning Screener

This assessment is intended to be administered to the whole class at once. A script is provided with detailed instructions, look-fors, and approximate timings.

The entire test will take 25-40 minutes depending on your students. There will likely be students who are unable to complete some of the tasks during the time provided. Plan to provide the time at the end of the assessment or at a later time for those students.

This assessment is different from state or other formal tests. Teachers are expected to engage with students, to re-read questions as necessary, answer questions, and encourage students to elaborate on their thinking. The purpose of the assessment is to gather as much evidence of student thinking as possible, and teachers are critical in this process.

When should this assessment be used? This assessment has been designed to be used at one of the three times below.

4th Grade

- Administer in the midst of fractions instruction as a formative assessment.
- Administer the assessment towards the end of instruction related to fractions to identify topics for re-teaching, additional practice, and targeted instruction.

5th Grade

- Use this assessment a few weeks before or early in the instruction of fractions in 5th grade as a readiness assessment and to identify areas for pre-teaching before fraction units of instruction begin.

Preparation

- Make copies of the assessment for all students.
- Have pencils with erasers (not pens) for all students.
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the video -- there is a video that accompanies this assessment for question 6. Have the video cued before you begin the assessment: linked here or access here: bit.ly/IntFRS-Q6.

Assessment Administration

Please follow the script as written and have a clock or timer ready. This will help to ensure that the assessment remains efficient. Students should not be aware of the timing of the tasks. This is not a timed test.

You may re-read questions.

Answer questions to ensure students understand the task. Refrain from answering questions that would influence student thinking or provide clues to how to solve the tasks.

Active Proctoring

Teachers are encouraged to move about the room and to interact appropriately and actively with the students.

- Where there are students who seem to not be engaging, it is appropriate to encourage them to re-read the questions to ensure understanding.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning.
- When you see student work that leaves you uncertain about their reasoning, ask questions and encourage them to draw, use number lines, or demonstrate their thinking in other ways.

Data Collection for Forefront® Users

Forefront has been configured to support data collection for Forefront® clients. Questions can be sent to support@forefront.education. Your account manager will support your district in account configuration and rostering to enable data entry by teachers.

Observations

While systemic collection of observational data is not included in this assessment, look-fors are called out in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring

Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the student to ensure the accuracy of the scoring and the analysis.

Accommodations

Any special accommodations which would normally be given for individual students should also be given for this assessment.

Pacing Guidelines

Suggested timings are provided for the tasks, however this is **not** a timed test. Give students enough time for each task so that it doesn't feel rushed, but also keep the test moving at a pace that keeps the students engaged and focused. Some students will likely need extra time after you have finished all the tasks to complete their work. It is recommended that teachers have a clock visible while giving the test in order to help pace the assessment, but students should not be made aware of this.

Script for the Administration of the Intermediate Fractional Reasoning Screener

Before You Begin the Assessment

- Read these instructions completely to familiarize yourself with the assessment.
- Prepare your classroom to minimize opportunities for students to see each other's work.
- Prepare to project the video to be shown for Task 6 (bit.ly/IntFRS-Q6).

How to Pace the Assessment

This assessment should take 25-40 minutes. To keep the students working at a good pace, suggested timings are provided for each question. Prepare a clock or timer before you start, but be discreet with it. This is not a timed test; the timings are provided as guidelines. Teachers are encouraged to use their best judgment as they lead the students through the assessment. Once almost all of the students have completed each task, continue to the next task and let students know that they will be provided with time to return to unfinished tasks later if necessary.

Questions may be re-read as necessary for the whole group, or for individual students.

The Script

Read all of the bolded text.

Teacher: **Today we are going to solve some problems to help me to understand how you understand fractions. The results of this assessment will not be part of your grade. This is only to help me to see how you understand fractions and to help me teach you better. For that reason it is important to always explain your answers clearly. Use words, drawings, number lines, and numbers to show your thinking.**

Please keep your eyes focused on your own paper. It is important that I get to know your own thinking about these ideas so that I can support you better. (Some teachers may want to spread students out a bit, or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please try not to work ahead, since these questions are a little different than other assessments. The last question will include a video that we will watch together.

If at any time you have a question, please raise your hand. If you need me to repeat a question, let me know. Are there any questions before we get started? *(Answer questions as necessary.)*

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

INTERMEDIATE

Task 1

Continue when all of the students are ready. Pause briefly after you read each of the numbers for Task 1 to give students the opportunity to write their numbers. Continue at a reasonable pace. You may say the numbers more than once if necessary.

Teacher: **For the first question I will read some numbers aloud. Write the numbers in each of the boxes.**
In box A write the number: one and one half.
In box B write the number: four sevenths.
In box C write the number: six thirds.

Repeat numbers as necessary to ensure every student has heard.

Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Look-Fors

If you see students writing words (like "one half"), redirect them to write it as a fraction.

Task 2, Part A

Teacher: **Find question 2a on your paper. 2a says, "This rectangle represents a piece of chocolate. This piece is three fifths of a whole chocolate bar.**

In the space below, draw the whole chocolate bar." Use lines, make marks, and show your thinking.

Clarify if necessary to ensure that the students understand the task.

Circulate the room to monitor progress, asking the students questions and encouraging them to show their thinking as necessary.

Questions to ask individual students as necessary:

- **Show me how you know yours is the right size.**
- For students who are hesitant to respond **- How big do you guess it would be?**

Look-Fors

- Are students gesturing and measuring with their fingers?
- Are students making marks on the gray rectangle?
- Are students partitioning the gray rectangle?

Pacing Guideline

about 2-3 minutes

Note

Do not provide, or allow students to use rulers. If a student asks to use a ruler or some other length measurement device, say something like, **That's not a bad idea, but just estimate for now. It does not need to be perfect.**

Task 2, Part B

Teacher: **I know some of you are still working, but let's take a look at Question 2b. 2b has some squares. "Write a fraction for the shaded portion of each square below." Write your fractions below or beside the square. You may draw on the squares to help you.** Ask students who finish early if they can write equivalent fractions for the shapes.

Look-Fors

- Students who are drawing on the squares to create equal portions.

Pacing Guideline

about 3 minutes

Note

Some students may protest that these don't look like fractions because the parts are not equal size. Be careful not to direct them! Ask simply, **"What fraction do you think it would be?"**

Task 2, Part C

Teacher: **Let's look together at question 2c. Turn the page and you will see question 2c. It says, "The gray shaded area shows one third stick of butter. Below are some more sticks of butter. Shade to show five thirds sticks of butter."**

Pacing Guideline

about 3 minutes

When most of the class is ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Task 3, Part A

Teacher: **Find question 3a. Question 3a says, "Place these numbers on the number line. Make marks and write the numbers to show the position of each number."**

When most students are ready proceed to part B.

Look-Fors

If students draw lines from the fractions to the number line instead of making marks and label them, make sure it is clear what they intend. Encourage students to write the numbers on the number line where they belong.

Pacing Guideline

about 1-2 minutes

Task 3, Part B

Teacher: **Look at question 3b. It says, "What fractions go in the boxes?" Look at the boxes below the line. Write the fractions that should go in each box." You may now work on both parts of question 3. Please raise your hand if you would like me to clarify the directions or repeat any of the questions I have already read.**

For students who finish quickly, ask if there are other possible fractions that they could put in the box on the right.

When everyone or almost everyone is ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Pacing Guideline

about 2-3 minutes

INTERMEDIATE

Task 4

Teacher: **Number four. Number 4a says, "The three dots below show one eighth of a pack of candies. Draw five eighths of a pack of candies in the box."**

Below you will see a line that says, "How many candies are in five eights of a pack?" Write the number of candies on the line.

Look-Fors

Look for students who only write an answer without showing any reasoning. Ask them to explain their thinking, or ask, "How do you know?"

Pacing Guideline

about 3 minutes for each part

After students have had a chance to answer, say, **Please look at question 4b. Question 4b says, "The dots below show three fourths of a pack of candies. How many candies are in a full pack?" Solve the problem, show your thinking and write the number of candies in the full pack on the line.**

Allow another 3 minutes for question 4 (or continue along earlier if everyone has finished). When ready, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Task 5, Part A

Teacher: **Question 5a says, "Jan says six tenths is equal to three fifths. Do you agree or disagree with Jan? Explain your answer. Include pictures or number lines in your explanation." You will be scored for your reasoning, not your answer, so make sure you explain why you agree or disagree.**

Look-Fors

Watch for students who only use numbers in their answers. Encourage them to show their reasoning using pictures (shapes) or number lines.

Pacing Guideline

about 3-5 minutes

Task 5, Part B

Teacher: **Turn the page and let's take a look at question 5b. Question 5b says, "Compare the following fractions using the greater than, equal to, or less than symbols. Explain your reasoning for each, using pictures, words, lines, and/or numbers." You will be scored for your reasoning, not your answers, so make sure you explain each comparison.**

Work on questions 5a and 5b now.

As students work on these tasks circulate through the room.

Allow 5 minutes for questions 5a and 5b (or continue along earlier if everyone has finished). After 5 minutes, say, **"We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more."**

Look-Fors

- When students do not justify their answer, ask how they can prove it.
- When students provide only numeric justifications for their reasoning, ask them if they have other ways that they can prove it.
- Where students use only drawings, ask if they have other ways they can show their reasoning. It is appropriate for students to also use words to explain their thinking.

Pacing Guideline

about 5 minutes

Task 6

Teacher: **Number 6. For this question I will read it, and then we will watch a short video that goes with the question. I will tell you when to start after we watch the video.**

"A rope is 2 feet long. Two thirds of a foot is cut off. How long is the remaining rope?"

Play the video (bit.ly/IntFRS-Q6 or linked here.)

Look-Fors

- Encourage students to explain their reasoning.
- Watch for students who convert the feet to inches. Encourage them to also write their answer in fractional form.
- Encourage students to write an equation on the line.
- If it is unclear, ask students to circle their final answer.

Note

If you see a student who answers problem 6 with inches (1 ft. 4 in.) ask them if they can also write their answer as a fraction.

Solve the problem. At the bottom, write an equation that matches your solution.

Give the students a few minutes to solve the rope problem. You may play the video more than once. Circulate the room.

Collect papers from students as they finish. As you do, check to see that their answers are complete. If there are incomplete papers, encourage students to finish those problems up, or to elaborate on their answers as necessary. Provide extended time to those who need it, or if necessary, separate those papers to allow those students to complete the work at a later time.

INTERMEDIATE

Name: _____ **Date:** _____

Show what you know about fractions on this assessment -- showing your thinking is as important as getting the answer.

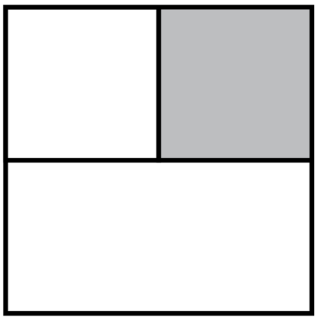
1. Write the numbers in each of the boxes as your teacher reads them.

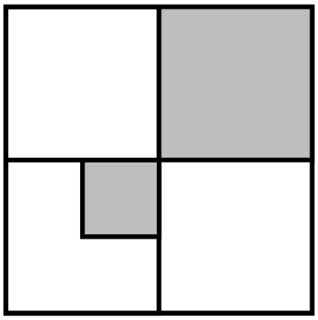
A	B	C

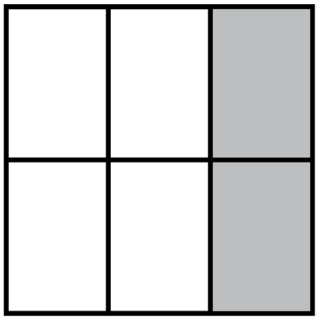
2. Part A: This rectangle represents a piece of chocolate. This piece is $\frac{3}{5}$ of the whole chocolate bar.

In the space below, draw the whole chocolate bar.

2. Part B: Write a fraction for the shaded portion of each square below.





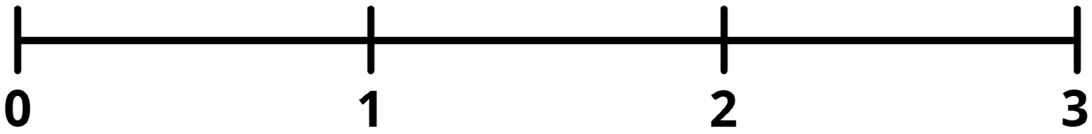


2. Part C: The gray shaded area shows $\frac{1}{3}$ of a stick of butter.

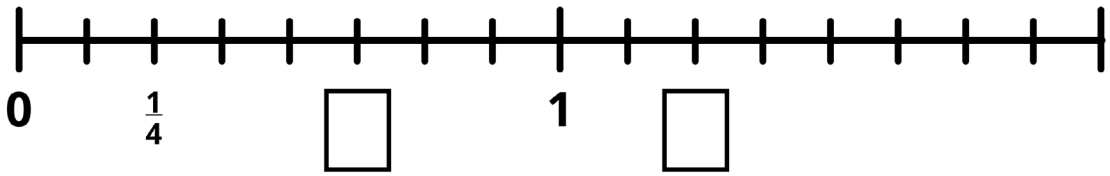
Below are some more sticks of butter. Shade to show $\frac{5}{3}$ sticks of butter.

3. Part A: Place these numbers on the number line. Make marks and write the numbers to show the position of each number.

$1\frac{1}{2}$ $\frac{2}{3}$

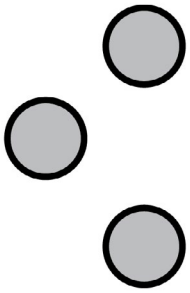


3. Part B: What fractions go in the boxes?



INTERMEDIATE

4. Part A: The 3 dots below show $\frac{1}{8}$ of a pack of candies. Draw $\frac{5}{8}$ of a pack of candies in the box.



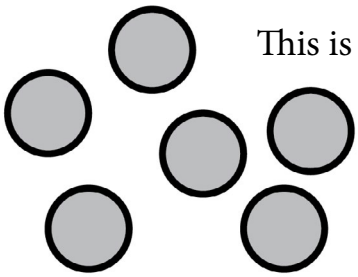
This is $\frac{1}{8}$ of a pack.

Draw $\frac{5}{8}$ of a pack here.

How many candies are in $\frac{5}{8}$ of a pack?

_____

4. Part B: The dots below show $\frac{3}{4}$ of a pack of candies. How many candies are in a full pack?



This is $\frac{3}{4}$ of a pack.

How many candies are in a full pack? _____

5. Part A: Jan says $\frac{6}{10}$ is equal to $\frac{3}{5}$. Do you agree or disagree with Jan? Explain your answer. Include pictures or number lines in your explanation.

5. Part B: Compare the following fractions using >, =, <. Explain your reasoning for each, using pictures, words, lines, and/or numbers.

$\frac{2}{5}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{6}{8}$ $\frac{8}{7}$ $\frac{7}{8}$

6. A rope is 2 ft. long. $\frac{2}{3}$ of a ft. is cut off. How long is the remaining rope?
Solve the problem below. Use words, pictures, lines, or numbers to explain your thinking.

Write an equation to match your solution. _____

INTERMEDIATE

Forefront Data Entry

Enter results as instructed in each of the questions.

Standards Alignments

At this time, these questions are simply aligned with the 4th grade domain of fractions - e.g. 4.NF. Specific alignments for different states will be done and configured in Forefront over time.

Fractional Reasoning Lens Facets

Alignments to the Fractional Reasoning Lens are listed with each of the tasks. For more on the lens, see the introduction of this document.

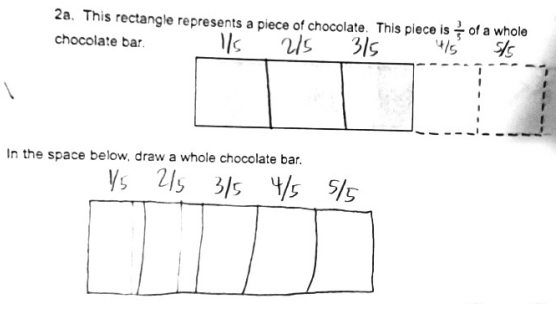
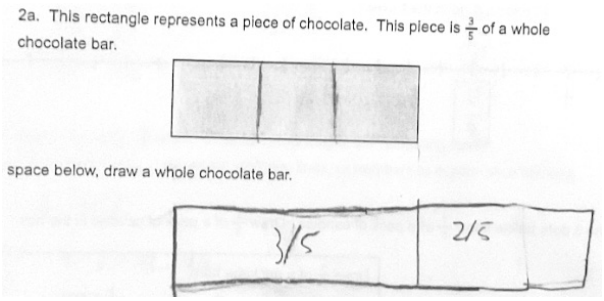
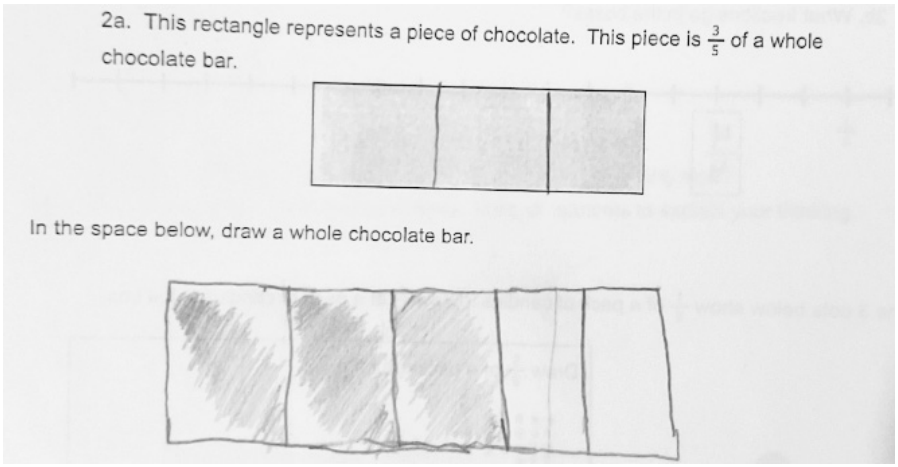
Observations

For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These may be taken into consideration during the scoring.

Task 1: Writing Fractions from Dictation

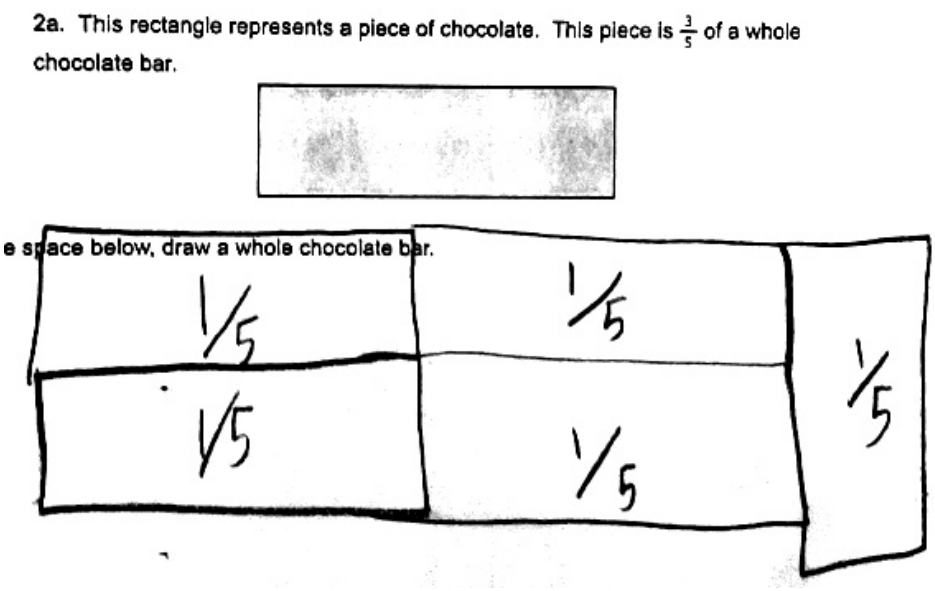
Facet	FR.4.WS (Words and Symbols)
Performance Levels	• Meeting (3 points) • Approaching (2 points) • Not yet (0-1 points)
Answer Key	A. $1\frac{1}{2}$ B. $\frac{4}{7}$ C. $\frac{6}{3}$ Give one point for each correctly written fraction.
Clarifications	Credit should be given if the number is legible. While students should be encouraged to write the numerals vertically, give points if they have written them like this: ½.
Readiness Expectation	Students should be able to write fractions and mixed numbers.

Task 2, Part A: Given 3/5 of a Rectangle, Show the Whole Rectangle

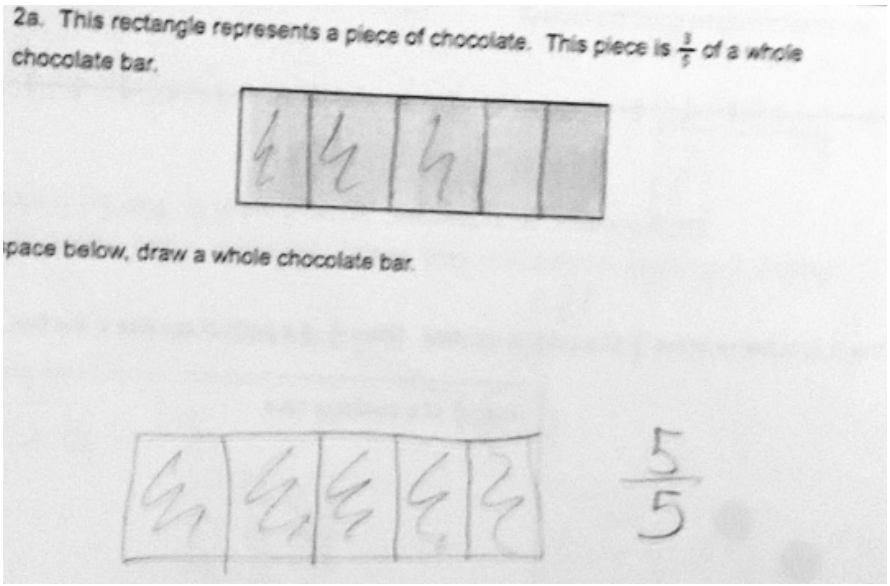
Facet	FR.4.Shapes (Shapes)
Performance Levels	• Shows reasoning (1 point) • Not yet (0 points)
Answer Key	  Give a 1 if the student accurately draws a rectangle that is two fifths larger than the gray rectangle and demonstrates reasoning through partitioning and iterating. The above samples represent clear reasoning. The original chocolate bar has been partitioned into 3 sections to find the unit fraction and then that unit has been iterated 5 times to represent the full chocolate bar.
Clarifications	Students will reason through this in a variety of ways, but the most common to look-for is when students have reasoned that they need to find the size of the unit fraction by dividing the "chocolate bar" into 3 pieces. To create the whole chocolate bar, the unit then needs to be repeated (iterated) 5 times. Precision is not of primary importance. Look for evidence of reasoning. While less clear, the student here has found the unit fraction and iterated it. This should be scored as "shows reasoning." 

Clarifications

Common misunderstandings: These solutions should be scored as "Not Yet." (0 points)

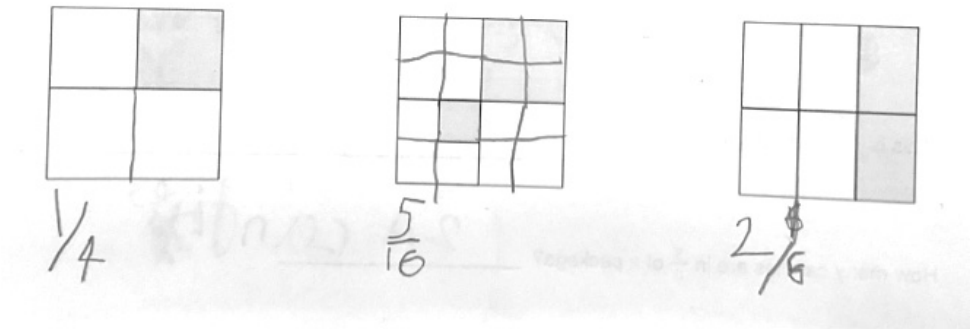
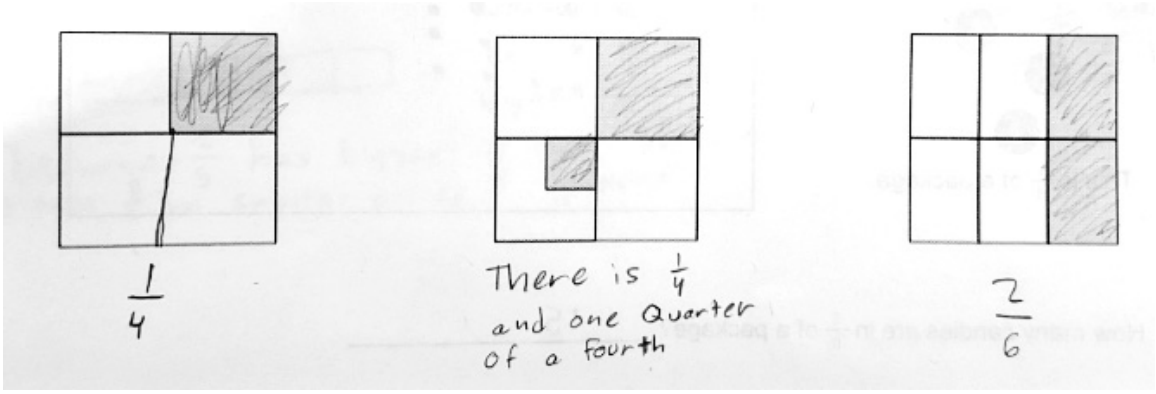


Notice on this next sample that the student has divided the original chocolate bar into 5 equal parts, thus their unit size is inaccurate, even if they have indicated that the whole bar would be longer.

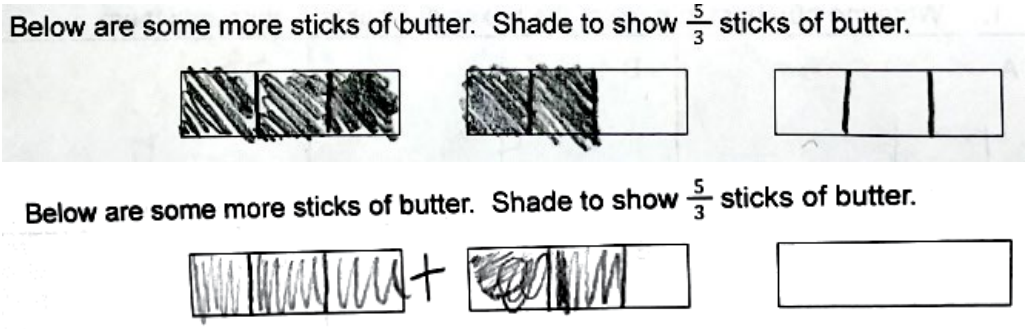


Note: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long. We can see that a student applies partitioning and iterating in their response without the need for precise measurements.

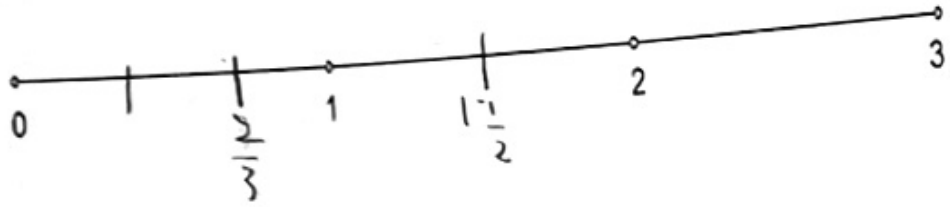
Task 2, Part B: Area of Squares: Reasoning About Equivalence

Facet	FR.4.Shapes (Shapes)
Performance Levels	• All correct - Meets expectations - Students understand and are able to create equal partitions in order to identify the fractional part of each shape. (3 points) • 1 or 2 correct - Approaching Expectations (1 - 2 points) • None correct - Not yet (0 points)
Answer Key	Give one point for each correct answer. $^1/_4$, $^5/_{16}$, $^1/_3$ or $^2/_6$ 
Clarifications	Accept equivalent fractions for any of the answers. However, if a student gives an unusual answer like $^2/_8$ instead of ¼, then ask the student to explain. While the second answer in the image below should be scored as incorrect, notice the brilliance of the answer this student has given. "There is ¼ and one quarter of a fourth." 
Readiness Expectation	Students know that fractions of geometric shapes need equal sized pieces. Students are able to subdivide shapes to create and write fractions to match. This question also helps to reveal those students who are able to readily recognize and generate equivalent fractions.

Task 2, Part C: 5/3 Sticks of Butter

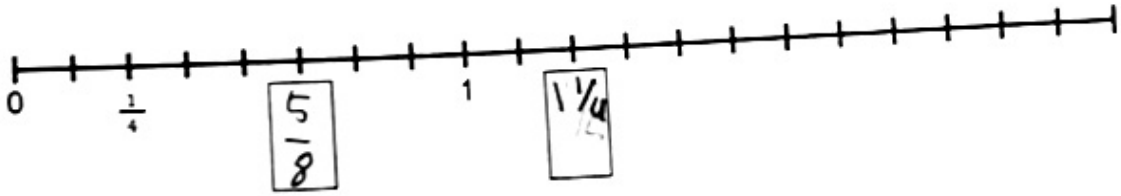
Facet	FR.4.Shapes (Shapes)
Performance Levels	• Meets Expectations - Students understand that unit fractions need to be iterated beyond a whole to create fractions that are greater than one (1 point) • Not Yet - 0
Answer Key	Students should have shaded one full rectangle and 2 more thirds, or any other combination to make 5 thirds. Below are two correct examples. 
Readiness Expectation	Students should know that the thirds need to be repeated 5 times to make a total of 5 thirds.

Task 3, Part A: Numbers on a Number Line

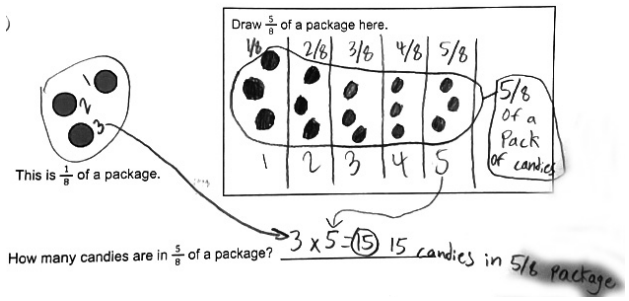
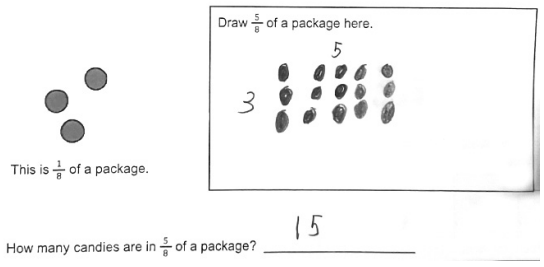
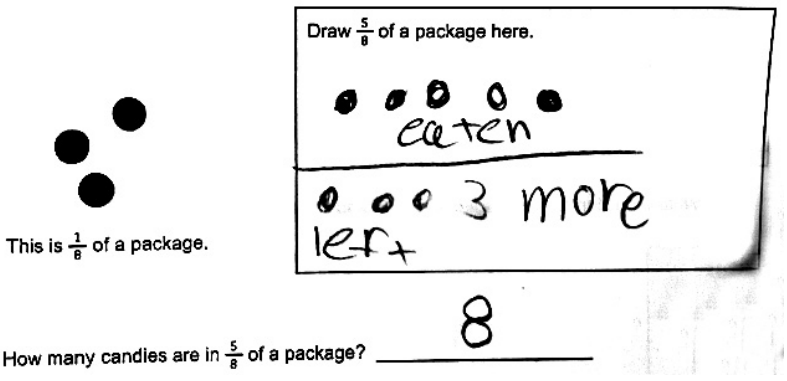
Facet	FR.4.NL (Number Lines)
Performance Levels	• Meets Expectations - Student demonstrates the ability to represent and locate both fractions on the number line. (2 points) • Approaching - Student has placed one of the numbers correctly on the line. (1 point) • Not yet - Student is beginning to learn how to represent and locate fractions on number lines. (0 points)
Answer Key	Give one point for each fraction if: 1 ½ is in the middle between 1 and 2. $^{2}/_{3}$ is right of center between 1 and 2. 

Clarifications	Note: Although problem 3a could be solved with a high degree of precision using a ruler, it would simply take too long.
Readiness Expectation	Students should be able to represent fractions on number lines that extend beyond 1. The importance of the line extending beyond 1 is that it forces the student to first consider the whole unit, and then partition it. The ability to accurately place fractions on lines demonstrates their ability to reason about the magnitude of the fraction relative to the unit. Watch for students who are reasoning by measuring with their fingers or pencils to divide the units into fractions. Students may make additional marks on the line to support their thinking. This is good evidence of their understanding that they need to break the whole numbers into the appropriate number of pieces and then, if necessary, iterate those to correctly place the fractions on the line.

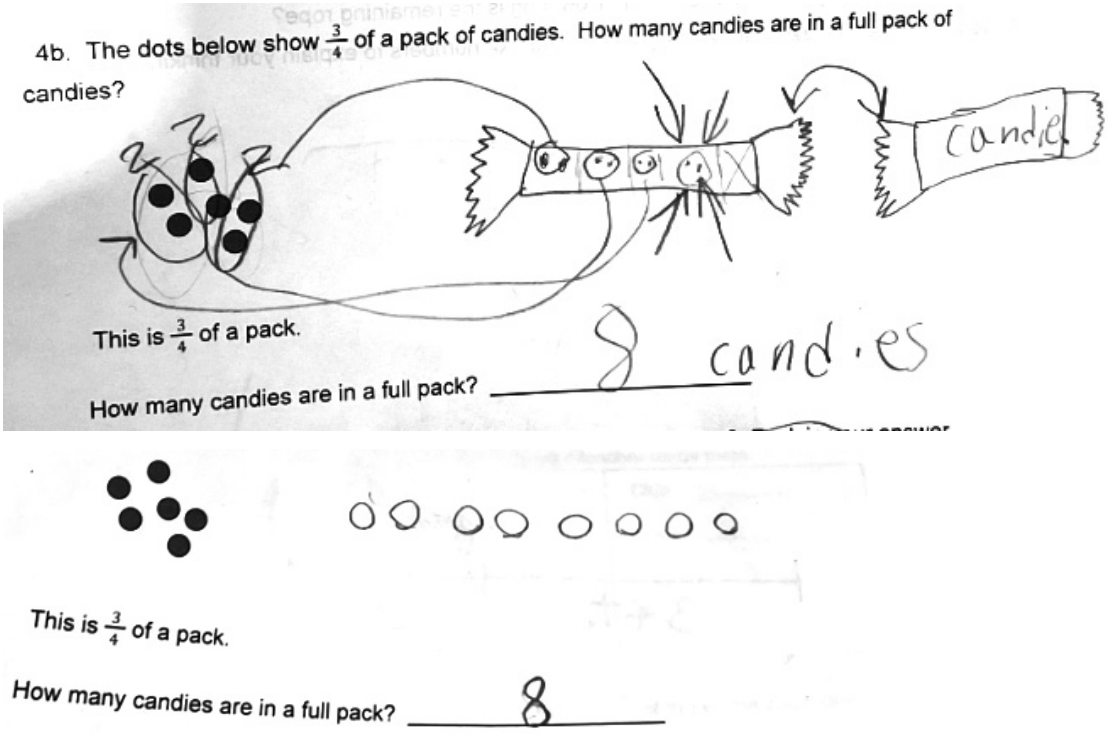
Task 3, Part B: Numbers on Number Line

Facet	FR.4.NL (Number Lines)
Performance Levels	• Meets Expectations - Student demonstrates the ability to represent and locate fractions on number lines. (2 points) • Approaching - Student is developing the ability to represent and locate fractions on number lines. (1 point) • Not yet - Student is beginning to learn how to represent and locate fractions on number lines. (0 points)
Answer Key	Give one point for each fraction if: • $^{5}/_{8}$ is in the box on the left. • $1^{1}/_{4}$ or $1^{2}/_{8}$ or $^{5}/_{4}$ or $^{10}/_{8}$ is in the box on the right. 
Readiness Expectation	This number line gets at two important ideas. The first is equal partitions. Students should recognize that there are 8 partitions between 0 and 1, and thus the first box is located at $^{5}/_{8}$. The second idea that this problem seeks to elicit is the idea of equivalent fractions. Students entering 5th grade should be able to use number lines to recognize and express equivalent fractions.

Task 4, Part A: One Eighth of a Pack

Facet	FR.4.Sets (Sets)
Performance Levels	• Meets Expectations - Student is able to apply understanding of the relationship of unit fractions to proper fractions as it relates to iterating sets. Correctly answers 15. (1 point) • Not yet (0 points)
Answer Key	Student answers 15.
Clarifications	Students should show 15 in the drawing. Students may arrange the dots in an array.   The incorrect example below shows the student does not yet understand how the unit of 3 candies needs to be iterated 5 times to show $^{5}/_{8}$. Notice that the student is seeing this as a subtractive problem. 
Readiness Expectation	This problem is about the ability to see a set as a unit fraction and to iterate that set to form a proper fraction. Students should be able to see that a set can represent a single unit that can be iterated. This problem presents an interesting crossover of fractional and whole number thinking. Students will often see the need to repeat the set of 3 five times, and then will calculate the answer using whole number thinking.

Task 4, Part B: From a Proper Fraction, Identify the Whole

Facet	FR.4.Sets (Sets)
Performance Levels	• Meets Expectations - Applies understanding of fractions to sets to find the whole. Correctly answers 8. (1 point) • Not yet (0 points)
Answer Key	Student answers 8 candies.
Clarification	Look for students to identify the unit of 2 candies and to add those two candies to the 6 that have been presented to see that a full pack would have 8 candies. 

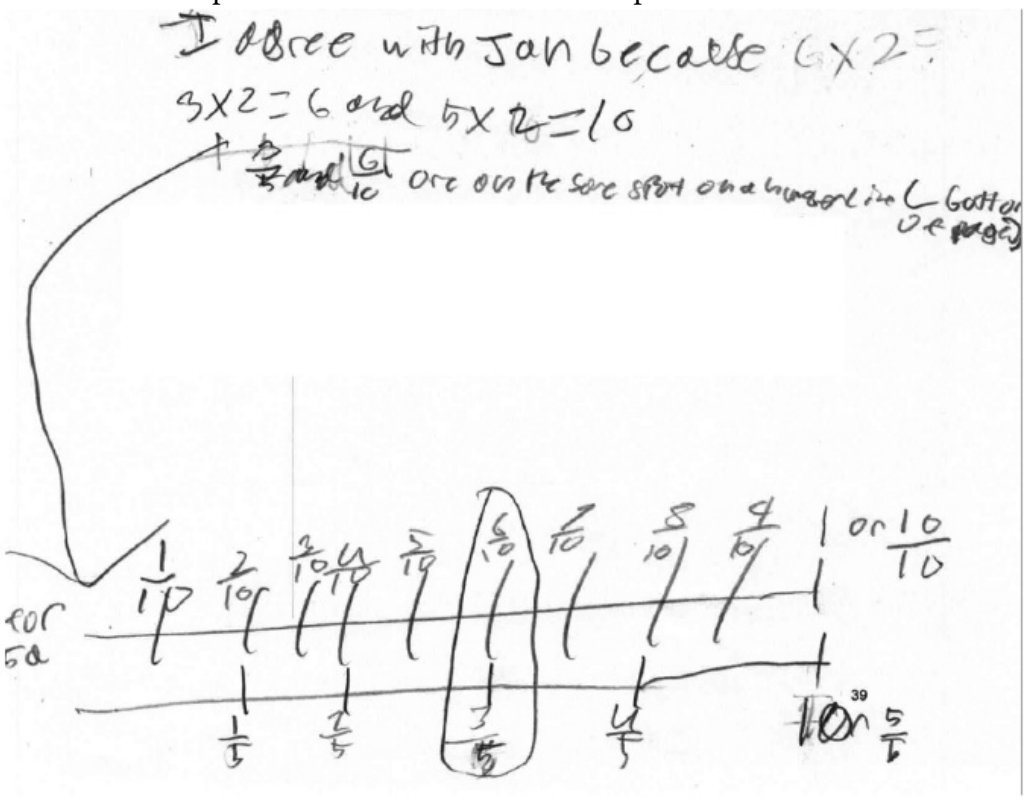
Task 5, Part A: Explain Equivalence

Facet	FR.4.MCE (Magnitude, Comparisons & Equivalence)
Performance Levels	• Meets Expectations - Student demonstrates the ability to clearly communicate why two fractions are equivalent. (1 point) • Not Yet - Student does not demonstrate an ability to represent a robust understanding of equivalent fractions. (0 points)
Answer Key	Give one point if the student agrees and demonstrates the ability to clearly communicate why the two fractions are equivalent. Read the clarifications for more information.

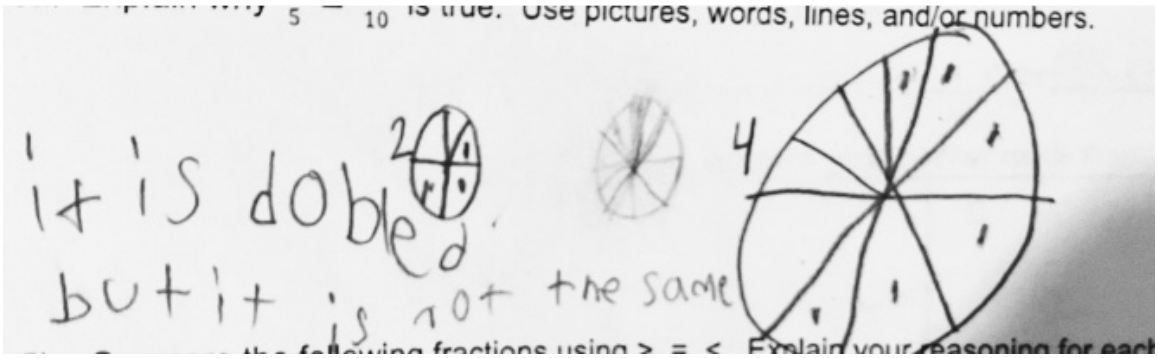
Clarifications

- Students who meet expectations demonstrate the ability to communicate the meaning of equivalent fractions using shapes, number lines, words, and/or numbers. If a student has only provided an equation (e.g. $^{3}/_{5} \times {}^{2}/_{2} = {}^{6}/_{10}$) ask if they can use a drawing or number line to explain why that is true.
- Look for students to show comparisons that show two equal sized units.
- If the student has not provided enough information (i.e. they simply agreed but gave no explanation) it is appropriate to ask them to elaborate before scoring.

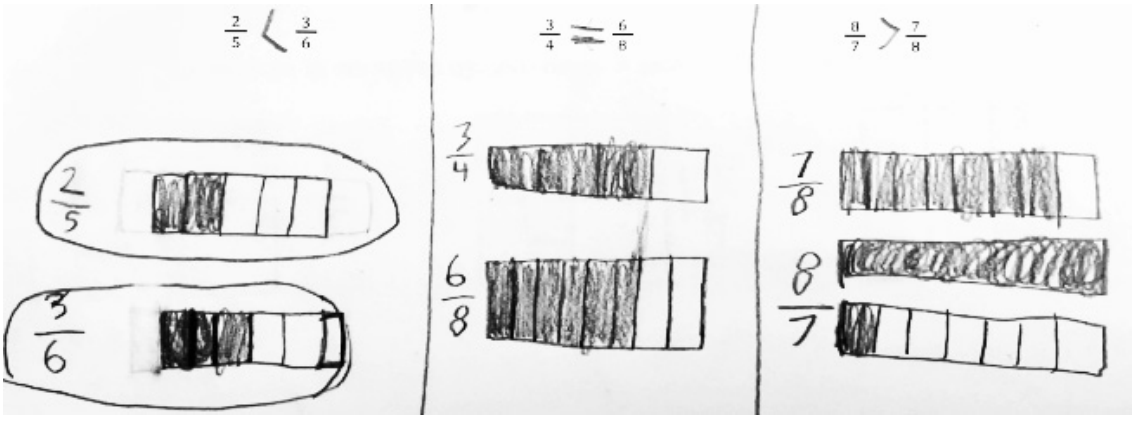
Below is an example of student work that meets expectations.



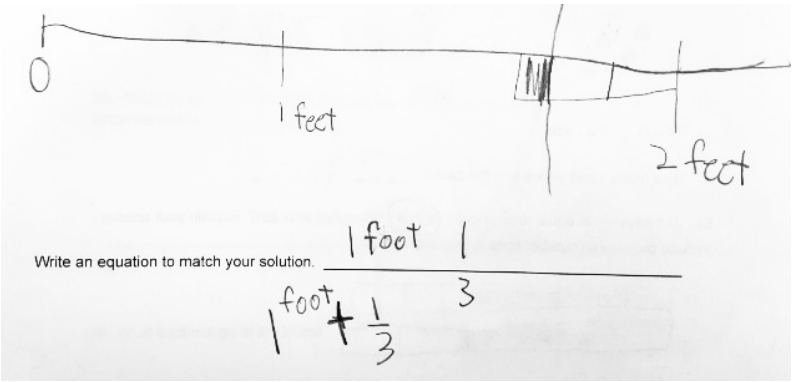
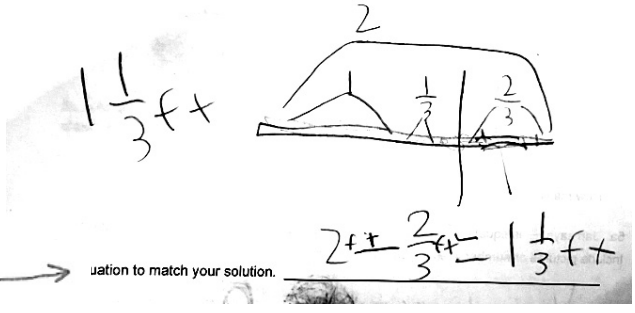
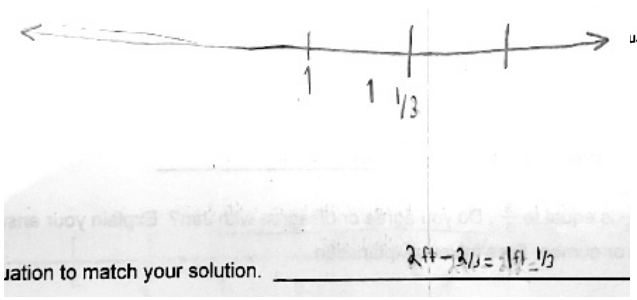
Look also for common misunderstandings: Notice how in the example below the student creates two fraction circles, one which is twice the size of the other.



Task 5, Part B: Compare

Facet	FR.4.MCE (Magnitude, Comparisons & Equivalence)
Performance Levels	• Meeting: Demonstrates the use of reasoning to compare basic fractions (3 points) • Approaching (2 points) • Not yet (<2 points)
Answer Key	Give one point for each correct fraction comparison and accompanying reasoning. Read the clarifications to better understand appropriate reasoning.
Clarification	• <u>It is not enough to simply put a comparison symbol.</u> If a student has only answered with a symbol, ask them to explain their answer with pictures, number lines, words, or numbers. • Give 1 point for each correct comparison with clear reasoning about the size and/or the equivalence of the fractions in order to compare them. Score the reasoning. If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<, >, =) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol, take note, and give a point for showing reasoning about size. • If the student is unable to reason about the size of the fractions, but instead applies some sort of procedure to correctly compare the fractions, ask the student if they have another way to show how they know their answer is correct. For example, if a student says that they know that 1/3 is greater than ¼ because the denominator is smaller, ask them to make a drawing to show how that works. • If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, or other means to explain their reasoning about the size of the fractions, <u>DO NOT give a point, even if the comparison is correct.</u> <u>NOTE:</u> If the student has provided evidence that is not sufficient for you to score the task with confidence, you may ask the student to make drawings, or ask the student to explain their thinking in order to inform your decision. 

Task 6: 2 ft. - 2/3 ft.

Facet	FR.4.C (Computation)
Performance Levels	• Meets Expectations: Student shows the ability to solve a simple subtraction problem with fractions and whole numbers in context, and write an equation that matches. (2 points) • Approaching: Student is either able to solve a problem or write an equation to match a problem, but not both. (1 point) • Not yet (0 points)
Answer Key	• Give one point for a correct solution. Student may draw pictures, or apply another strategy to solve the problem. Accept either $1\frac{1}{3}$ or $\frac{4}{3}$ as a correct answer. • Give one point for a correct equation: $2 - \frac{2}{3} = 1\frac{1}{3}$ (Credit can be given also for a partially correct equation (e.g. $2 - \frac{2}{3} = 1\frac{1}{2}$). In this case the student correctly sees the calculation the problem solves for, but is incorrect in their computation. The sample below should receive one point for a correct answer, but no point for an equation.  Below are two examples that should receive 2 points. One for a correct answer, one for the correct equation.  

Instructions for the Administration of the Advancing Fractional Reasoning Screener

This assessment is intended to be administered to the whole class at once. A script is provided with detailed instructions, look-fors, and approximate timings.

The entire test will take 35-50 minutes depending on your students. There will likely be students who are unable to complete some of the tasks during the time provided. Plan to provide the time at the end of the assessment or at a later time for those students.

This assessment is different from state or other formal tests. Teachers are expected to engage with students, to re-read questions as necessary, answer questions, and encourage students to elaborate on their thinking. The purpose of the assessment is to gather as much evidence of student thinking as possible, and teachers are critical in this process.

When should this assessment be used? This assessment has been designed to be used at one of the times below.

5th Grade

- Administer in the midst of fractions instruction as a formative assessment.
- Administer the assessment towards the end of instruction related to fractions to identify topics for re-teaching, additional practice, and targeted instruction.

6th Grade

- Use this assessment a few weeks before or early in the instruction related to fractions in 6th grade as a readiness assessment and to identify areas for pre-teaching before fraction units of instruction begin.

Preparation

- Make copies of the assessment for all students.
- Have pencils with erasers (not pens) for all students.
- Take appropriate precautions to ensure that students are not looking at one another's papers.
- Cue the videos -- There are two videos that accompany this assessment for questions 6a (https://bit.ly/AdvFRS-Q6a) and 6b (https://bit.ly/AdvFRS-Q6b). Have the videos cued before you begin the assessment.

Assessment Administration

Please follow the script as written and have a clock or timer ready. This will help to ensure that the assessment remains efficient. Students should not be aware of the timing of the tasks. This is not a timed test.

You may re-read questions.

Answer questions to ensure students understand the task. Refrain from answering questions that would be leading, influence student thinking, or provide clues to how to solve the tasks.

Active Proctoring

Teachers are encouraged to move about the room and to interact with the students.

- When students are not engaging, encourage them and re-read the questions to ensure understanding.
- When you see student work that leaves you uncertain about the reasoning, ask questions and encourage them to write, draw, use number lines, or demonstrate their thinking in other ways.
- For students who respond quickly and then disengage, encourage them to elaborate on their thinking and to explain their reasoning. See the script for look-fors and additional prompts to consider during the assessment.

Data Collection for Forefront® Users

Forefront has been configured to support data collection for Forefront® clients. Questions can be sent to support@forefront.education. Your account manager will support your district in account configuration and rostering to enable data entry by teachers.

Observations

While systemic collection of observational data is not included in this assessment, look-fors are called out in the scoring guide. Teachers are encouraged to carry a clipboard to record observations.

Scoring

Included with each assessment is a detailed rubric for scoring. When the evidence on the page is inconclusive as to whether the student has demonstrated reasoning (beyond the application of mere procedures) teachers should discuss the responses with the student to ensure the accuracy of the scoring and the analysis.

Accommodations

Any special accommodations which would normally be given for individual students should also be given for this assessment.

Pacing Guidelines

Suggested timings are provided for the tasks, however this is **not** a timed test. Give students enough time for each task so that it doesn't feel rushed, but also keep the test moving at a pace that keeps the students engaged and focused. Some students will likely need extra time after you have finished all the tasks to complete their work. It is recommended that teachers have a clock visible while giving the test in order to help pace the assessment, but students should not be made aware of this.

Script for the Administration of the Advancing Fractional Reasoning Screener

Before You Begin the Assessment

- Read these instructions completely to familiarize yourself with the assessment.
- Prepare your classroom to minimize opportunities for students to see each other's work.
- Prepare to project the videos to be shown for Task 6A (https://bit.ly/AdvFRS-Q6a) and 6B (https://bit.ly/AdvFRS-Q6b).

How to Pace the Assessment

This assessment should take 35-50 minutes. To keep the students working at a good pace, suggested timings are provided for each question. Prepare a clock or timer before you start, but be discreet with it. This is not a timed test; the timings are provided as guidelines. Teachers are encouraged to use their best judgment as they lead the students through the assessment. Once almost all of the students have completed each task, continue to the next task and let students know that they will be provided with time to return to unfinished tasks later if necessary.

Questions may be re-read as necessary for the whole group, or for individual students.

The Script

Read all of the bolded text.

Teacher: **Today we are going to solve some fraction problems so I can learn more about how you understand fractions. The results of this assessment will not be part of your grade. This is only to inform my teaching and to help me teach you better. For that reason it is important to always explain your answers clearly. Use words, drawings, number lines, and numbers to show your thinking.**

Please keep your eyes focused on your own paper. It is important that I get to know your own thinking about these ideas so that I can support you better. (Some teachers may want to spread students out a bit, or provide barriers as appropriate to ensure that each student's work represents their own independent thinking.)

I'll read each of the questions as we work through the assessment. Please do not work ahead, since these questions are different than other assessments. The last question will include two videos we will watch together.

As you are solving the problems there will be space for you to make drawings and number lines and write words. Please show your thinking as well as you can to explain your answers and justify your answers.

If at any time you have a question, please raise your hand. If you need me to repeat a question, let me know. Are there any questions before we get started? *(Answer questions as necessary.)*

Distribute the papers.

Teacher: **Please write your name and the date on the page.**

ADVANCING

Task 1

Continue when all of the students are ready. Pause briefly after you read each of the numbers for Task 1 to give students the opportunity to write their numbers. Continue at a reasonable pace. You may say the numbers more than once if necessary.

Teacher: **For the first question I will read some numbers aloud. Write the numbers in each of the boxes.**
In box A write the number: one and one third.
In box B write the number: three tenths.
In box C write the number: eight thirds.

Look-Fors

If you see students writing words (like "one third"), redirect them to write it as a fraction.

Repeat numbers as necessary to ensure every student has heard. Once the students have had sufficient opportunity to write the numbers, proceed to Task 2.

Task 2, Part A

Teacher: **Now let's take a look at 2a. "For each of the shapes below write a fraction for the shaded part." Begin.**

Questions to ask individual students as necessary:

- For students who finish quickly, ask, **Are there other fractions that would also be correct for any of these?** (This is to see if they can generate equivalent fractions, especially for the two rectangles on the right.)

Allow 2 minutes for this question (continue along earlier if everyone has finished). Say, **We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more.**

Look-Fors

- Students who are drawing on the squares to create equal portions.

Note

Some students may protest that some of these don't look like fractions because the parts are not equal size. Be careful not to direct them! Ask simply, **"What fraction do you think it would be?"**

Pacing Guideline

about 2 minutes

Task 2, Part B

Teacher: **"This rectangle represents a piece of chocolate. This piece is two fifths of the whole chocolate bar. In the space below draw a whole chocolate bar."**

You may make marks to help you. If you want to, you can also draw on the gray rectangle to show your thinking.

Look-Fors

- Are students drawing on the original chocolate bar?
- For students who draw a chocolate bar without clearly indicating their thinking, ask, **How do you know your chocolate bar is the right size? How can you show your answer is correct?**

Pacing Guideline

about 2-3 minutes

Note

Do not provide, or allow students to use, rulers. If a student asks to use a ruler or some other length measurement device, say, **That is not a bad idea, but just estimate for now. It does not need to be perfect.**

Task 2, Part C

Teacher: Once most students have completed question 2b, say: **Look down now to task 2c. Task 2c says, "The rectangle below represents one fourth of a stick of butter. You need three half sticks of butter for a recipe. Add to the rectangle to make three half sticks of butter." Begin.**

When ready, say, **We are going to continue to the next task. If you have not finished, don't worry, I will give you time later to think about it some more.**

Look-Fors

- Are students adding equal sized pieces to the quarter bar that is already there?
- Do some students divide the quarter piece in half (showing confusion of the relationship between quarters and halves)?

Pacing Guideline

about 2 minutes

Task 3, Part A

Teacher: **"Here is a number line. It has the numbers zero to five already on it. Put these numbers onto the number line."** (Do not read the fractions.) **Make a mark on the line where each number belongs, and write the number by each mark.**

Circulate the room to ensure that students are clearly marking their number lines and, if necessary, remind them to write the fraction to indicate which fraction goes with which mark.

Look-Fors

- Where students put fractions in places and it is unclear why, it is okay to ask, **How do you know it goes there?** to gather more formative information.

Pacing Guideline

about 2 minutes

After about 2 minutes say, **We are going on to the next question. If you need more time, I will give you time to come back to this question later.**

Task 3, Part B

Teacher: **Number 3b. "The line below has zero and one already marked on it. There are also some fractions, but they are missing either their numerators or denominators. Put numbers into the boxes to complete the fractions so that they make sense."**

Pacing Guideline

about 3 minutes

Task 4, Part A

Teacher: **Number 4a. "The circles below show a package of ten candies. I want to give one fifth of the package of candies to my friend. Show how many of the candies I want to give my friend."**

How many candies are in one fifth of the package? Show your thinking, and write your answer on the line.

Remind students to write their numeric answer in the space provided if they have not.

As the students finish, say, **We are going on to the next question. If you need more time I will give you time to come back to this question later.**

Look-Fors

- Students may either draw to show how many candies, or may indicate by drawing on the box of candies.
- For students who quickly write only the answer, encourage them to show their thinking.
- For students who answer quickly and accurately, ask if they can write an equation to match the situation.

Pacing Guideline

about 3 minutes

Task 4, Part B

Teacher: **Number 4b. "Joseph has a collection of cars. He puts 8 of his cars on the table and says, 'This is two thirds of my collection.' How many cars are in his full collection?" Show your thinking and write your answer on the line provided.**

Look-Fors

- Circulate the room to encourage students to explain their reasoning in the space provided and/or by drawing on the image.

Pacing Guideline

about 3 minutes

Task 5, Part A

Teacher: **Let's look together at the next question. Number 5a. "Use greater than, equal to, and less than signs to compare each pair of fractions. Explain your answers with drawings, number lines, and/or words."**

It is important on this question to show your thinking clearly. I will be scoring your reasoning more than your answers. So, be sure to prove how you know each of your comparisons is correct.

Circulate and observe students working.

Pacing Guideline

about 5 minutes

Look-Fors

- Encourage them to make drawings for each comparison.
- Look for rectangles, number lines, or verbal explanations.
- Students may also want to use numbers to help them make the comparisons. This is good too, but not sufficient. Ask the students to show their thinking with number lines or drawings.
- When students have applied a procedure like finding common denominators, ask them to confirm their answers are correct with drawings or number lines.
- Ask students to explain their thinking.

We are going to continue to the next question. If you are still working, I will provide some time for you to finish.

Task 5, Part B

Teacher: **Number 5b. "Write three fractions that are equivalent to four sixths. Show your thinking in numbers, words, or pictures."**

Pacing Guideline

about 3 minutes

Look-Fors

- For students who quickly find equivalencies, ask them to show how this works.

Note

This question will be scored by correct answers only, so while drawings and number lines can be helpful, they are not as critical on this question as they were with the previous question.

Task 6, Part A

Teacher: **Number 6a. For this question I will read it, and then we will watch a short video that goes with the question. I will tell you when to start after we watch the video.**

"A strip of paper is two and one-half inches long. I cut off three fourths of an inch. How much of the strip is left? Solve in the space below and circle your answer.

Below it says, "Write an equation to match your solution."

Look-Fors

- If students only solve numerically, encourage them to explain their thinking.
- Remind students to write an equation if necessary.

Pacing Guideline

about 4-5 minutes

Let's watch the video. Play the video (https://bit.ly/AdvFRS-Q6a). (The video can be shown more than once if necessary.)

After the video say**, Solve the problem in the space provided, circle your answer, and write an equation that goes with this problem.**

Task 6, Part B

Teacher: **Number 6b. For this question I will read it and then we will watch another short video that goes with the question. I will tell you when to start after we watch the video.**

"I have 3 cups of rice. I am using a one-fourth cup to measure servings onto plates. I want to use all of the rice. How many plates will I fill with one-fourth cup each? Solve in the space below and circle your answer."

Below it says, "Write an equation to match your solution."

Look-Fors

- If students only solve numerically, encourage them to explain their thinking.
- Remind students to write an equation if necessary.

Pacing Guideline

about 5 minutes

Let's watch the video. Play the video (https://bit.ly/AdvFRS-Q6b). (The video can be shown more than once if necessary.)

After the video say**, Solve the problem in the space provided, circle your answer, and write an equation that goes with this problem.** Give time to answer the question. Then say, **That is the last question. Take some time now to look over your answers and complete any questions you haven't finished.**

Encourage students to review their assessments and check their work. Conclude the assessment. As you collect the papers ensure that all the questions are answered and ask students to finish things as necessary. (Look in particular at problem 5a, and encourage students to make drawings to justify their comparisons.) If there are students who need more time, provide it now, or later.

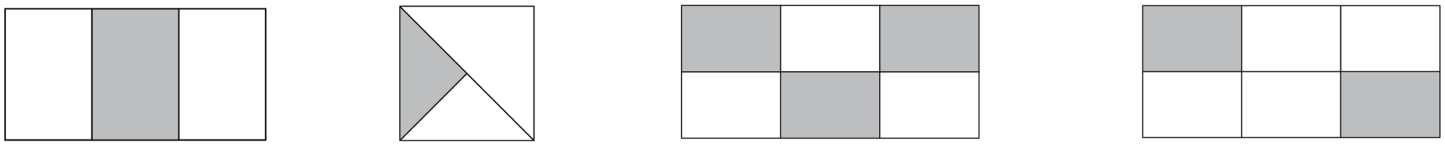
ADVANCING

Name: _____ **Date:** _____

Show what you know about fractions on this assessment -- showing your thinking is as important as getting the answer.

1. Write the numbers in each of the boxes as your teacher reads them.

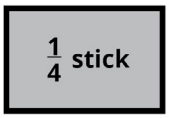
2. Part A: For each of the shapes below write a fraction for the shaded part.



2. Part B: This rectangle represents a piece of chocolate. This piece is $\frac{2}{5}$ of the whole chocolate bar.

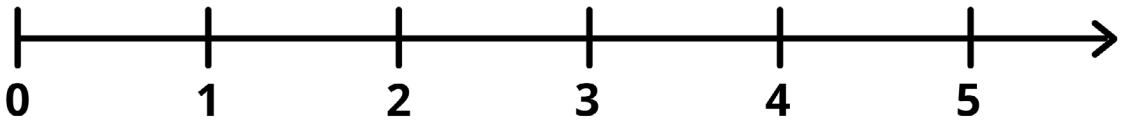
In the space below draw a whole chocolate bar.

2. Part C: The rectangle below represents $\frac{1}{4}$ of a stick of butter. You need $\frac{3}{2}$ sticks of butter for a recipe. Add to the rectangle to make $\frac{3}{2}$ sticks of butter.

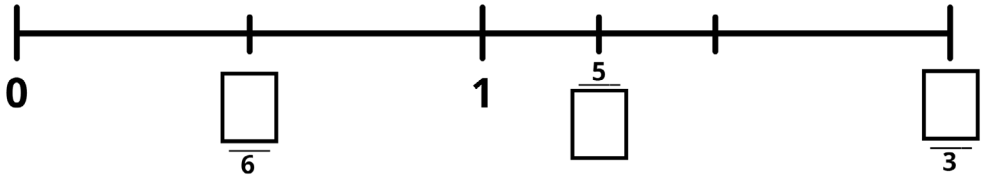


3. Part A: Here is a number line. It has the numbers 0 - 5 already on it.
Put these numbers onto the number line.

$\frac{1}{3}$ $\frac{2}{3}$

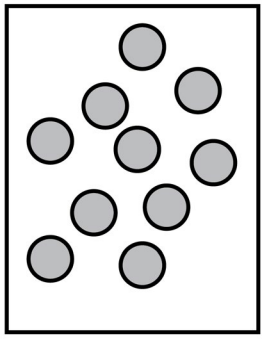


3. Part B: The line below has 0 and 1 already marked on it. There are also some fractions, but they are missing either their numerators or their denominators. Put numbers into the boxes to complete the fractions so that they make sense.



4. Part A: The circles below show a package of 10 candies. I want to give $\frac{1}{5}$ of the package of candies to my friend. Show how many of the candies I want to give my friend.

10 candies



How many candies are in $\frac{1}{5}$ of the package? _____

4. Part B: Joseph has a collection of cars. He puts 8 of his cars on the table and says, "This is $\frac{2}{3}$ of my collection." How many cars are in his full collection? Show your thinking.

Joseph's full collection has _____ cars.

5. Part A: Use >, =, < to compare each pair of fractions. Explain your answers with drawings, number lines, and/or words.

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{3}{2}$ $\frac{2}{3}$ $\frac{2}{6}$ $\frac{3}{9}$

5. Part B: Write three fractions that are equivalent to $\frac{4}{6}$.
Show your thinking in numbers, words, or pictures.

6. Part A: A strip of paper is $2\frac{1}{2}$ inches long. I cut off $\frac{3}{4}$ inch. How much of the strip is left? Solve in the space below and circle your answer.

Write an equation to match your solution. _____

6. Part B: I have 3 cups of rice. I am using a $\frac{1}{4}$ cup to measure servings onto plates. I want to use all of the rice. How many plates will I fill with $\frac{1}{4}$ cup each? Solve in the space below and circle your answer.

Write an equation to match your solution. _____

Forefront Data Entry

Enter results as instructed in each of the questions.

Standards Alignments

At this time, these questions are simply aligned with the grade-level domain of fractions - e.g. 4.NF. Specific alignments for different states will be done and configured in Forefront over time.

Fractional Reasoning Lens Facets

Alignments to the Fractional Reasoning Lens are listed with each of the tasks. For more on the lens, see the introduction of this document.

Observations

For some tasks, students' demonstration of reasoning will happen while they solve the tasks. Assessors should be alert and take notes of the behaviors they see. These may be taken into consideration during the scoring.

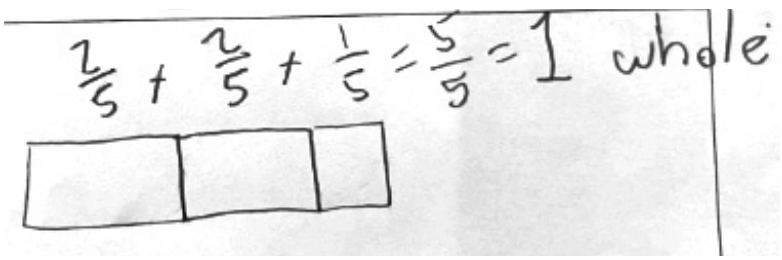
Task 1: Writing Fractions from Dictation

Facet	FR.5.WS (Words and Symbols)
Performance Levels	• All accurate (3 points) • Not yet (< 3 points)
Answer Key	A. $1\frac{1}{3}$ B. $\frac{3}{10}$ C. $\frac{8}{3}$ Give one point for each correctly written fraction.
Clarifications	Credit should be given if the number is legible and accurate. While students should be encouraged to write the numerals vertically, give points if they have written them like this: $1\frac{1}{3}$.
Readiness Expectation	All correct.

Task 2, Part A: Fractions of Rectangles

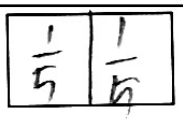
Facet	FR.5.Shapes (Shapes)
Performance Levels	• All correct - Meets expectations (4 points) • Approaching expectations (2-3 points) • Not yet (0-1 points)
Answer Key	Give one point for each correctly written fraction.  ⅓ ¼ ½ or 3/6 2/6 or ⅓
Readiness Expectation	Students need to recognize that fractions need to have equal sized pieces, and that those pieces do not need to be contiguous. This task represents a low level of understanding and is intended to identify students and groups of students who would benefit from review. Some students will write the equivalent fractions - while this is not considered in the scoring, when students do this, it provides important formative information.

Task 2, Part B: Given 2/5 of a Rectangle, Show the Whole

Facet	FR.5.Shapes (Shapes)
Performance Levels	• Shows accurate reasoning - student accurately draws a rectangle that is two fifths larger than the gray rectangle and demonstrates partitioning and iterating. (1 point) • Not yet (0 points)
Answer Key	Give 1 point for a correct solution.
Clarification	Students may reason through this in a variety of ways, but the most common to look for is when students have reasoned that a total of five fifths need to be constructed. Students should show a recognition of the unit of $\frac{1}{5}$ as half the size of the given rectangle to create the whole rectangle the fifths then need to be repeated (iterated) 5 times. Precision is not of primary importance. Look for evidence of reasoning. When possible, ask students to explain their thinking if it is not obvious. 

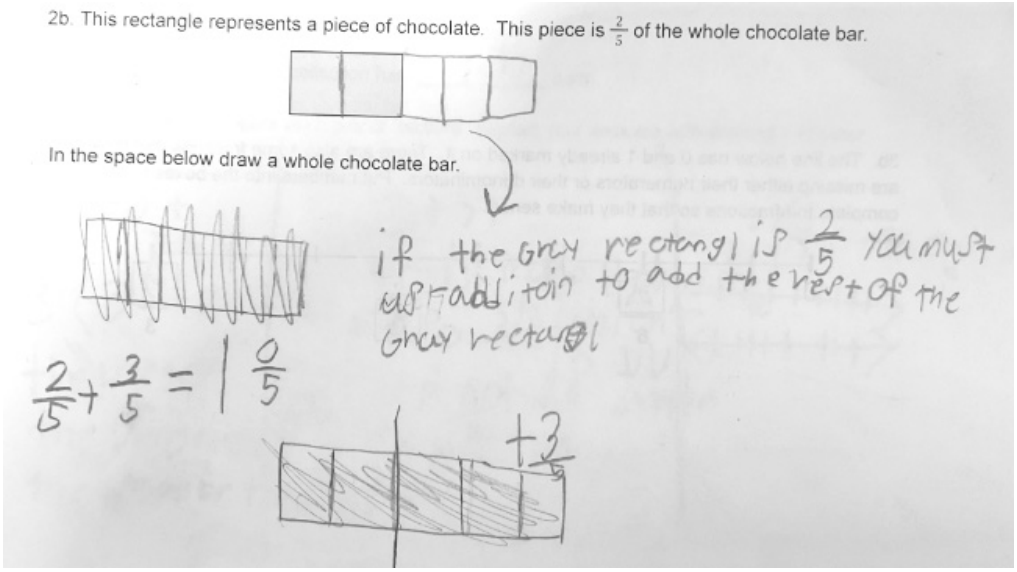
Clarification

Here is another example of a correct solution.

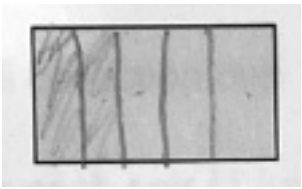


In the space below draw a whole chocolate bar.

One more example of a correct solution.

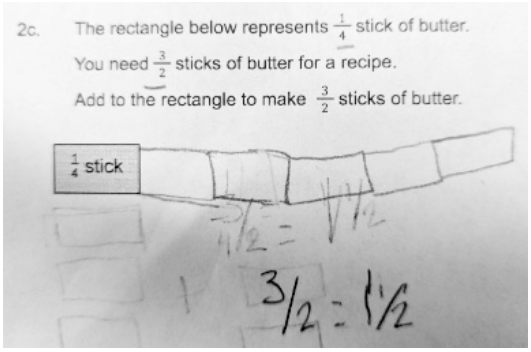
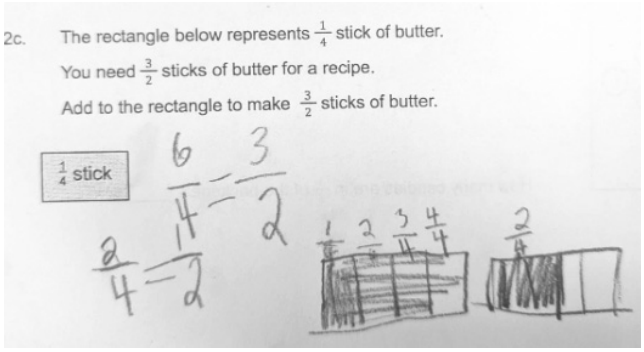
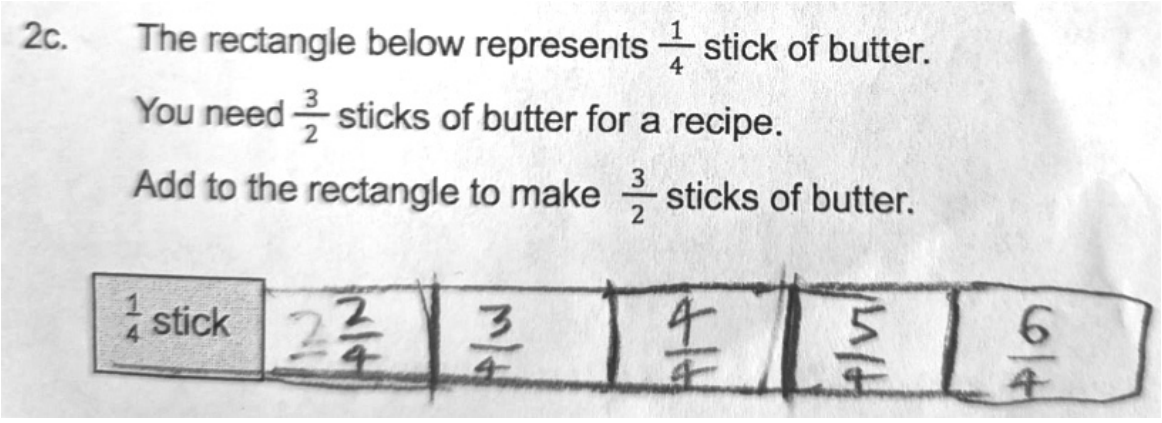


A common misunderstanding to watch out for: Many students will want to divide the chocolate into 5 equal pieces, not recognizing that in this situation the numerator indicates the number of partitions necessary to find the unit fraction. The example to the right is not accurate and should be scored 0 (not yet).

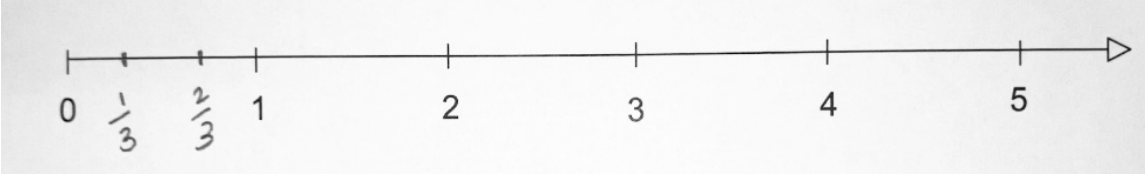


Note: Although this problem could be solved with a high degree of precision using a ruler, it would simply take too long. We can see that a student applies partitioning and iterating in their response without the need for precise measurements.

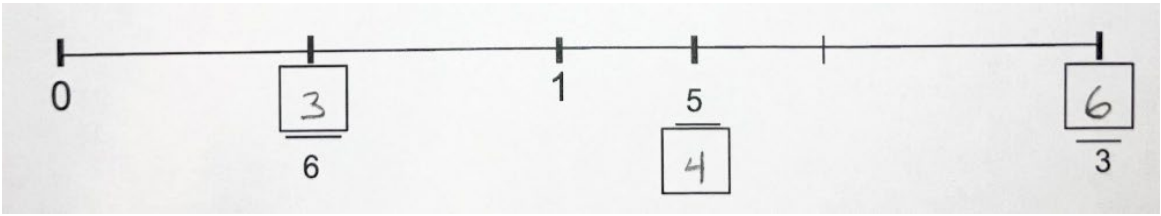
Task 2, Part C: Three Halves of Butter

Facet	FR.5.Shapes (Shapes)
Performance Levels	• Meets Expectations (1 point) • Not Yet (0 points)
Answer Key	Give 1 point for a response that clearly shows that the quarter length from the sample needs to be iterated 6 times to build 3 halves (or one and one half).
Clarification	The examples below should be scored as "Meets Expectations."   Notice how these two example not only shows the students understanding that two quarters equals one half, but also that ³⁄₂ is equivalent to 1 ½. The below example, while not making clear the connection of ³⁄₂ to 1½ makes clear the connection between ⁶⁄₄ and ³⁄₂. 
Readiness Expectation	Students show that they understand that two quarters are needed to make a half. This is a simple equivalency that students should be able to demonstrate using the geometric shape (a rectangle). Additionally, students should demonstrate that the halves need to be iterated 3 times to make the fraction requested. Students may show that they have created one whole followed by one more half, but this is not necessary for getting full credit on this question.

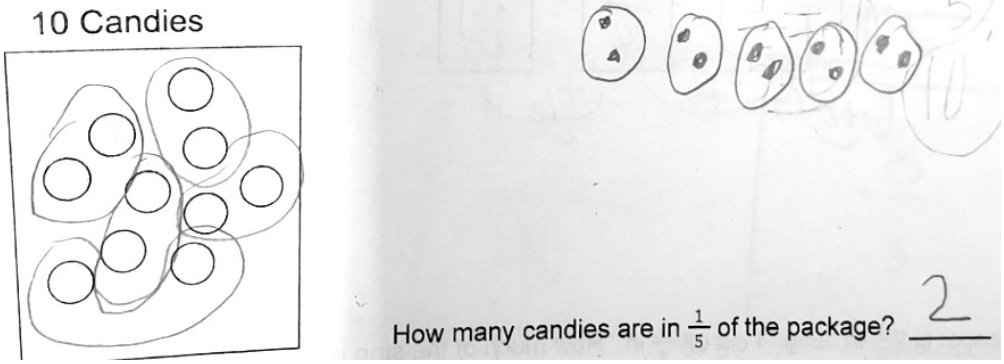
Task 3, Part A: Fractions on Number Lines

Facet	FR.5.NL (Lines)
Performance Levels	• Meets Expectations (2 points) • Approaching (1 point) • Not yet (0 points)
Answer Key	Give one point for each correctly placed fraction. Give a point for ⅓ if it is placed left of center between 0 and 1. Give a point for ⅔ if it is placed right of center between 0 and 1. 
Readiness Expectation	Students should be able to place fractions on number lines with reasonable accuracy. The distraction of having the line to 5 allows for students to reveal their understanding of fractions as being relative to whole numbers on number lines. If a student has placed the fractions in incorrect positions, ask them to explain their thinking for formative purposes.

Task 3, Part B: Missing Numerators and Denominators

Facet	FR.5.NL (Lines)
Performance Levels	• Meets Expectations (3 points) • Approaching (2 points) • Not yet (0-1 points)
Answer Key	Give one point for each correct response. 
Readiness Expectation	Students should be able to apply their understanding of the meaning of numerators and denominators to reason about what must be the missing part of each of these fractions.

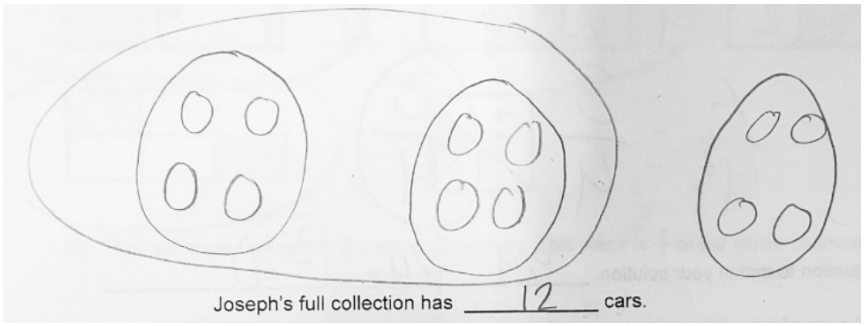
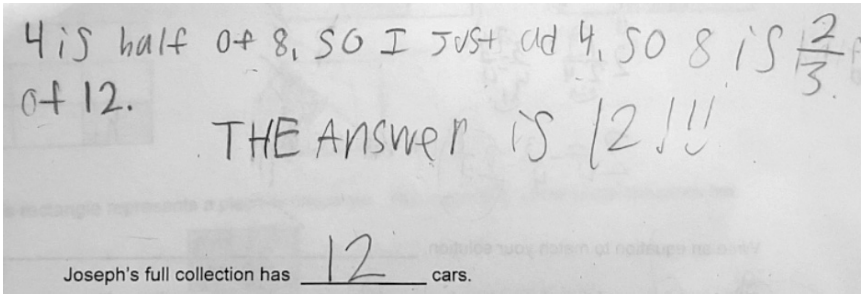
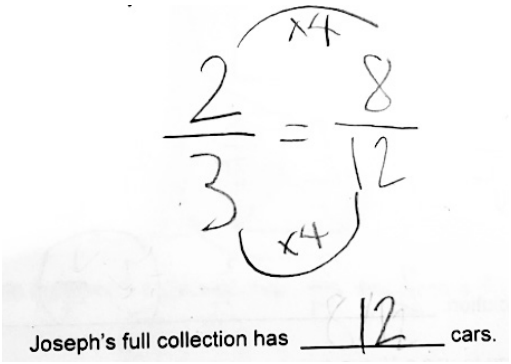
Task 4, Part A: 1/5 of a Package

Facet	FR.5.Sets (Sets)
Performance Levels	• Meets expectations - Student is able to divide the set into 5 equal groups and correctly respond that 2 candies would be ⅕ of the package (1 point) • Not Yet - Student does not identify that 2 candies is ⅕ of the set (0 points)
Answer Key	Give one point for a correct answer (2). 
Readiness Expectation	This question, if tied to an expression, would be ⅓ * 6 or 6 * ⅓ which aligns with 5th grade expectations for fractions in most states. However, the ability to understand that generating fractions means creating equal partitions aligns with 3rd grade expectations. To be fully ready for the work of 6th grade students should be able to identify a fraction of a set, and also to connect this understanding to multiplication.

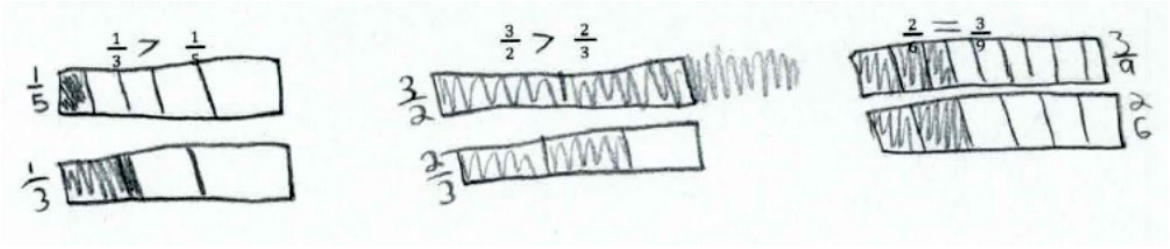
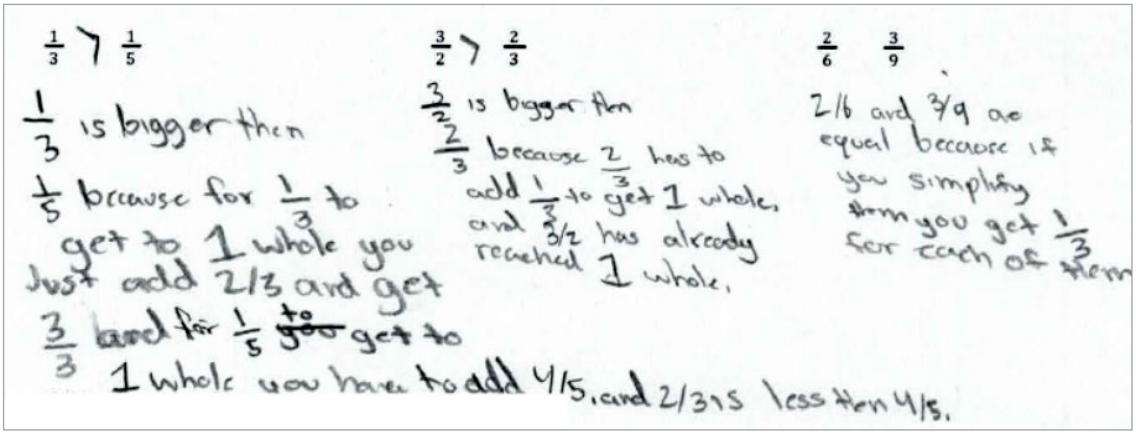
Task 4, Part B: 8 is two thirds. How much is 1?

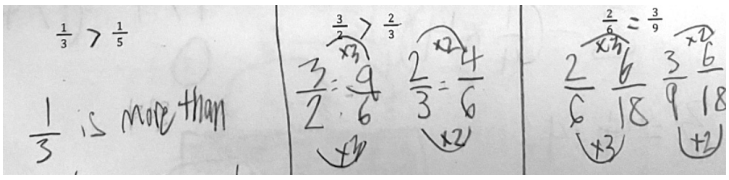
Facet	FR.5.Sets (Sets)
Performance Levels	• Meets Expectations - Correctly answers 12 (1 point) • Not yet (0 points)
Answer Key	Give 1 point for a correct answer (12). Encourage students who only put a numeric answer to explain their reasoning in the space provided, but only the answer will be used in the scoring of this task. Read on for clarification.

ADVANCING

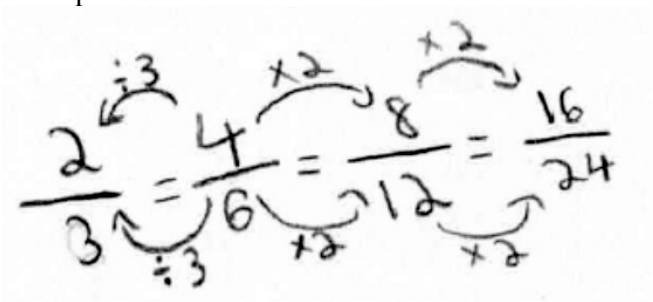
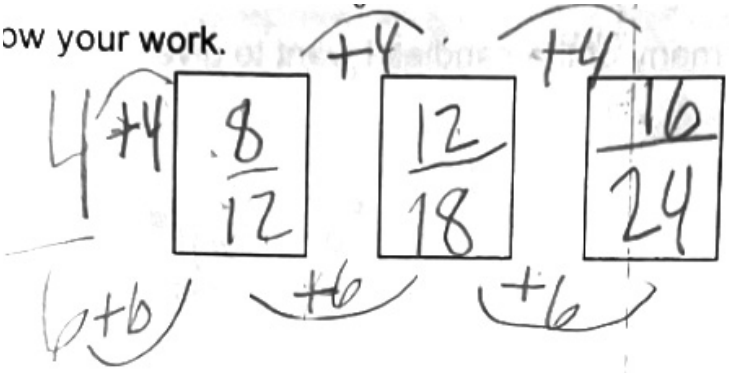
Clarification	Here are two examples of work that demonstrates reasoning.   Notice that this third example (below) of a correct response really represents a student demonstrating proportional reasoning. 
Readiness Expectation	Students who are ready for the work of 6th grade have an understanding of fractions that allows them to understand that the 8 cars need to be divided into equal parts as defined by the numerator in ⅔, and that one more part needs to be added to create the whole. Students are not expected to solve this problem with an equation at this time, though the situation could be modeled with the equation 8 ÷ ⅔. If the student does use this equation, challenge them to make a visual representation to match.

Task 5, Part A: Find Equivalent Fractions

Facet	FR.5.MCE (Magnitude, Comparisons & Equivalence)
Performance Levels	• Meets Expectations - Student fully demonstrates an ability to reason about the size of fractions (beyond or alongside applying a procedure) in order to compare them (3 points) • Approaching (2 points) • Not Yet - Student does not fully demonstrate an ability to reason about the size of fractions to compare them (0-1 points)
Answer Key	• For each comparison, give 1 point for each correct comparison with clear and correct reasoning about the size and/or the equivalence of the fractions in order to compare. • If a student has made a drawing that appears to correctly show the comparisons, but the comparison symbol (<, >, =) does not match, ask the student to explain. If an incorrect comparison is the result of confusion with the symbol, direct the student to correct the symbol, take note, and give the point for showing reasoning about size. • If the student applies some sort of procedure to correctly compare the fractions (cross multiplication, or by finding common denominators) ask the student if they have another way to compare them, or ask them to use a drawing or number lines to show how they know their answer is correct. For example, if a student says that they know that ⅓ is greater than ⅕ because the denominator is smaller, ask them to make a drawing or use a number line to show how that works. • If the student is unable to demonstrate the accuracy of their comparisons using drawings, words, number lines, numbers, or other means to explain their reasoning about the size of the fractions <u>DO NOT give points, even if the comparison is correct.</u> The two samples below should be scored as 3:  

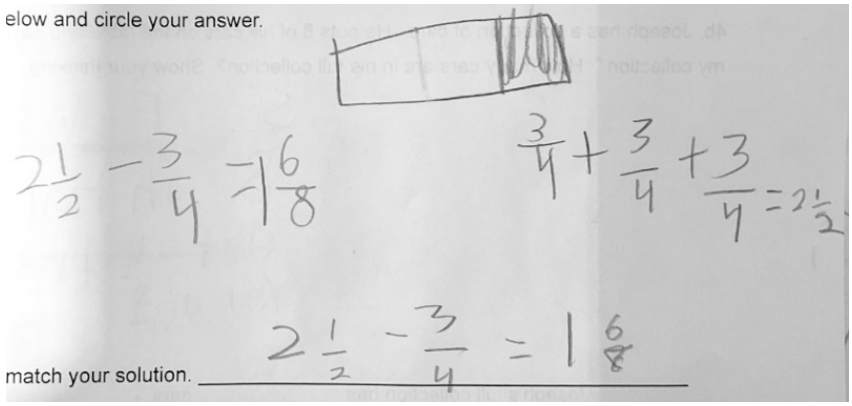
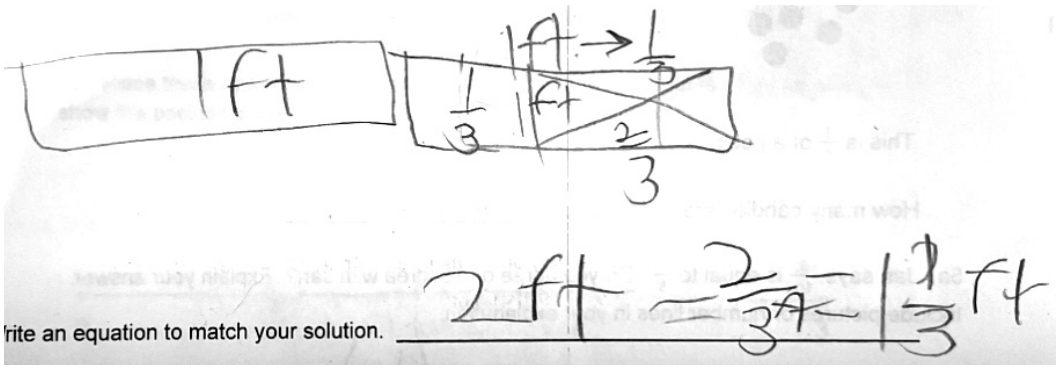
Clarification	This sample to the right cannot be reliably scored as a 3. The teacher should return to the student to ask them to provide more evidence, or talk to the student to ensure that they are able to reason about these comparisons beyond using the procedure demonstrated. 

Task 5, Part B: Find Equivalent Fractions

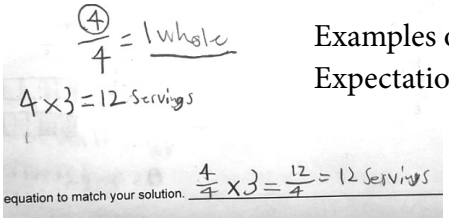
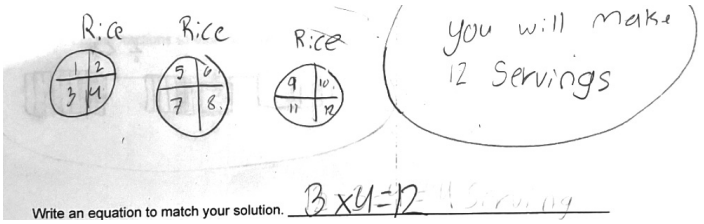
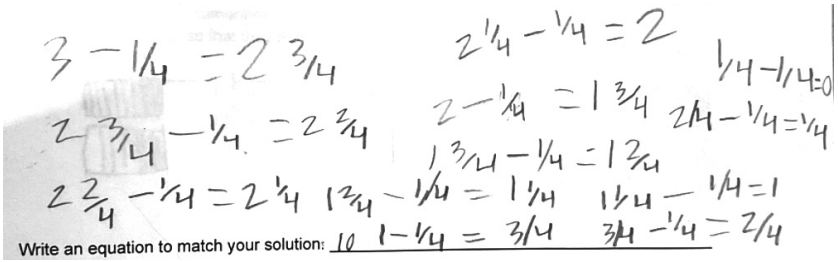
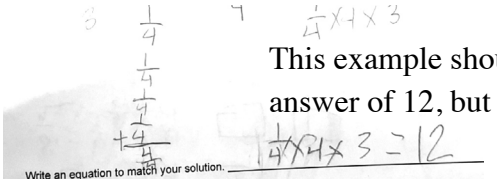
Facet	FR.5.MCE (Magnitude, Comparisons & Equivalence)
Performance Levels	• Demonstrates the ability to generate equivalent fractions (3 points) • Approaching (2 points) • Not yet (0-1 points)
Answer Key	Give one point for each accurate equivalent fraction. Examples are $^{2}/_{3}$, $^{4}/_{6}$, $^{6}/_{9}$, $^{8}/_{12}$, $^{10}/_{15}$, $^{20}/_{30}$ Examples are:  While this the answer below is correct, and should be scored as such, notice that the student appears to be applying additive rather than multiplicative reasoning in finding the equivalent fractions. This should be noted for instructional purposes. 

ADVANCING

Task 6, Part A: 2 1/2 inches - 3/4 inches

Facet	FR.5.C (Computation)
Performance Levels	• Meets Expectations - Student shows the ability to solve a simple subtraction problem with fractions and whole numbers in context, and write an equation that matches (2 points) • Approaching Expectations - Student is either able to solve a problem or write an equation to match a problem, but not both. (1 point) • Not yet (0 points)
Answer Key	• Give one point for a correct solution. Accept either 1 ¾, ⁷⁄₄ or an equivalent (e.g. 1⅝) as a correct answer. • Give one point for a correct equation: 2 ½ - ¾ = 1¾, or some variation thereof. Below are examples of correct solutions (2 points)  

Task 6, Part B: 2 Cups Divided into Quarter Cups

Facet	FR.5.C (Computation)
Performance Levels	• Meets Expectations - Shows ability to solve problem and create equation to match (2 points) • Approaching Expectations - Student is either able to solve a problem or write an equation to match a problem, but not both (1 point) • Not yet - Solution and equation both incorrect (0 points)
Answer Key	Two points possible: Give one point for a correct solution of 12. Give one point for a correct equation: $3 \div \frac{1}{4} = 12$ or $3 \times 4 = 12$ or a series of equations (see the fourth example below).
Clarification	 Examples of responses that should be scored as Meets Expectations (2). Notice the variety of equations students use. This solution, that uses whole numbers in its equation, should be scored 2. Correct.  While it would have been good to ask this student to elaborate on their thinking with a drawing or something to demonstrate their thinking, this next response (below) should also be scored as Shows Reasoning (2).  This response below, though not fully correct (they appear to have answered 10) they have shown that this problem can be solved with repeated subtraction and have created a series of equations to do so, and so should be scored as Approaching Expectations (1).   This example should be given 1 point since the student has a correct answer of 12, but an inaccurate equation.